1. The radioactive element ^{238}U decays via the emission of an alpha particle (a ^{4}He nucleus, symbol α). What is the other decay product? How much energy is released? All this energy is in the form of the kinetic energy of the decay products. How fast is the alpha particle travelling?

A ^{4}He nucleus has 2 neutrons and 2 protons while ^{238}U, an isotope of Uranium is element 92, and thus has 92 protons and 238 – 92 = 146 neutrons. The decay products are found by conserving neutrons and protons. The other decay particle will have 92 – 2 = 90 protons, so it is element 90 which is Thorium. The particular isotope will have 146 – 2 = 142 neutrons. The isotope number is the total number of nucleons, protons and neutrons, = 90 + 142 = 238 – 4 = 234. The element is ^{234}Th. The decay is thus $^{238}U \rightarrow {}^{234}Th + {}^{4}He$.

From the text, Appendix A, the masses are 238.050784 u, 234.043593 u, and 4.002602 u respectively. The mass difference between the sides is Δm = (238.050784 u) – (234.043593 u + 4.002602 u) = 4.589×10^{-3} u. This missing mass goes into the kinetic energy of the particles. The amount of kinetic energy is

$$\Delta m = 4.589 \times 10^{-3}\ \text{u} \times 931.49432\ \text{Mev}/c^2/\text{u} = 4.2746\ \text{MeV}/c^2.$$

To find the speed we note that momentum must be conserved. If the ^{238}U was initially at rest

$$\gamma(u_T)m_T u_T = \gamma(u_\alpha)m_\alpha u_\alpha\ . \qquad (1)$$

Also the energies must be conserved

$$\gamma(u_T)m_T c^2 + \gamma(u_\alpha)m_\alpha c^2 = \gamma(0)m_U c^2\ . \qquad (2)$$

Now $\gamma(0) = 1$ since the uranium atom is at rest. However, these are messy equations and are messy to solve algebraically. Solving numerically we find $u_T = 0.000812$ c and u_α = 0.047447 c.

(In this problem, I have not taken into account the fact that electrons do not accompany the alpha particle which is He^{++} as a result. This does not change the result.)

2. A neutron outside the nucleus β decays into a proton, an electron, and a neutrino. Note that n = 1.008665 u, p = 1.007285 u, and e = 5.48578×10^{-4} u.

(a) Assuming the neutrino is massless, how much energy is released?

(b) Assuming that all this energy is converted into the kinetic energy of the electron, how fast is the β electron moving?

(c) If you ignored relativistic effects, how fast would the electron be moving.

(a) The β decay reaction is $n \rightarrow p + e + \nu$. The difference in mass between the left and right-hand sides of the reaction is converted into energy according Einstein's formula, $E = (\Delta m)c^2$. The mass difference is

$$\Delta m = 1.008665 \text{ u} - 1.007285 \text{ u} - 0.000549 \text{ u} = 0.000831 \text{ u} .$$

Converting to energy, this is

$$\Delta m = 0.000831 \text{ u} \times 931.5 \text{ MeV}/c^2 = 0.7741 \text{ MeV}/c^2.$$

So $E = \Delta m\, c^2 = 0.7741$ MeV.

(b) The relativistic kinetic energy is $E_k = \gamma(u)mc^2 - mc^2 = [\gamma(u)-1]mc^2$. The rest energy of the electron is 0.5110 MeV. So $\gamma(u) - 1 = 0.7741/0.5110$ and we need to solve for u. First $\gamma(u) = 2.5149$. Squaring both sides and inverting yields $1 - (u/c)^2 = 0.1581$. Thus $u/c = 0.9175$ or $u = 2.751 \times 10^8$ m/s.

(c) For the non-relativistic case, $E_k = ½mu^2$. Solving for u, we get

$$u/c = [2E/mc^2]^{½} = [2(0.7741)/(0.5110)\ ]^{½} = 1.7406$$

or $u = 5.218 \times 10^8$ m/s. This value exceeds the speed of light.

3. Beta decay can occur inside a nucleus with the proton remaining inside the daughter product while the electron and neutrino escape. As can be seen from Appendix A of Tipler, ^{11}Be decays this way. Write out the decay formula. Assuming that the neutrino is massless, determine the recoil speeds of the decay products.

The radioactive element ^{11}Be had 11 nucleons, 4 protons and 7 neutrons. After the beta decay one neutron becomes a proton, so we still have 11 nucleons but now there are 5 protons and 6 neutrons. The element is ^{11}B. Thus the reaction is

$$^{11}\text{Be} \rightarrow {}^{11}\text{B} + e + \nu \,.$$

The difference in mass between the left and right-hand sides of the reaction is converted into energy according Einstein's formula, $E = (\Delta m)c^2$. The mass difference is

$$\Delta m = 11.021657 \text{ u} - 11.009305 \text{ u} - 0.000549 \text{ u} = 0.011803 \text{ u} \,.$$

Converting to energy, this is

$$\Delta m = 0.000831 \text{ u} \times 931.49401 \text{ MeV/c}^2 = 10.9944 \text{ MeV/c}^2.$$

So $E = \Delta m\, c^2 = 10.994$ MeV.

Let's assume the ^{11}Be was at rest. Conservation of momentum yields

$$\gamma(u_T)m_T u_T = \gamma(u_\alpha)m_\alpha u_\alpha \,. \tag{1}$$

Also the energies must be conserved, assuming the neutrino has negligible energy,

$$\gamma(u_B)m_B c^2 + \gamma(u_e)m_e c^2 = \gamma(0)m_{Be}c^2 \,. \tag{2}$$

Now $\gamma(0) = 1$ since the ^{11}Be atom is at rest. However, these are messy equations and are messy to solve algebraically. Solving numerically we find $u_B = 1.120 \times 10^{-3}$ c and $u_e = 0.9990$ c.

4. Consider ^{31}P which has 15 protons and 16 neutrons. Find the binding energy per nucleon. Note that n = 1.008665 u, ^{1}H = 1.007825 u, and ^{31}P = 30.973762 u.

The binding energy is the energy difference between the atom ^{31}P and its nucleons separately. It appears as a mass difference between the two. The binding energy is then given by Einstein's formula, $E = (\Delta m)c^2$. The mass difference is

$$\Delta m = 15(1.007825 \text{ u}) + 16(1.008665 \text{ u}) - 30.973762 \text{ u} = 0.282253 \text{ u} .$$

Converting to kg, this is

$$\Delta m = 0.282253 \text{ u} \times 931.5 \text{ MeV/c}^2\text{/u} = 262.92 \text{ MeV/c}^2.$$

Dividing this by the number of nucleons (15 + 16 = 31), we find

$$E_{\text{per nucleon}} = 262.92 \text{ MeV} / 31 = 8.48 \text{ MeV}$$

5. Can ^{14}C decay into ^{12}C through the spontaneous emission of two neutrons?

The decay would be

$$^{14}C \rightarrow {}^{12}C + n + n.$$

Referring to Appendix A of Tipler we see ^{14}C = 14.003242 u, ^{12}C = 12.000000 u, and n = 1.008665 u. The total mass on the right-hand side is 14.017330 u. So energy would need to be supplied to the ^{14}C; it would not occur spontaneously.