

1. The probability of spontaneous transitions of a charged particle from one initial energy level to another final level is governed by the overlap integral

$$\bar{D}_o = q \int \vec{r} \psi_i^* \psi_f dV,$$

where q is the charge of the particle. If $\bar{D}_o = 0$ between two levels, the transition is forbidden. Searching for such forbidden transitions leads to *selection rules*. Consider the 3D infinite potential well.

- (a) Show that only allowed transitions from $\psi_{n1,n2,n3}$ are to a state where two of the quantum numbers are unchanged and the other quantum number changes by an odd number like 1, 3, 5, etc.
- (b) Show that the emitted photons have energies $E = p(2n - p)E_1$ where the n is the original quantum level and $n - p$ is the final quantum level and E_1 is the ground state energy of the one-dimensional potential well.

The following integral identities are useful:

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \frac{L}{2} \delta_{nm} \quad \text{and} \quad \int_0^L x \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} \frac{L^2}{4} & n = m \\ \frac{2L^2}{\pi^2} \frac{nm[(-1)^{n-m} - 1]}{(n-m)^2(n+m)^2} & n \neq m \end{cases}$$

2. For the hydrogen atom prove the selection rules $\Delta m = 0$ or ± 1 and $\Delta l = \pm 1$. Remember, in spherical coordinates $\vec{r} = ir \sin \theta \sin \phi + jr \sin \theta \cos \phi + kr \cos \theta$. Make use of symmetry

where possible. The identities $\int_0^{2\pi} e^{-ib\phi} d\phi = \delta_{b0}$, $\sin \phi = \frac{e^{i\phi} - e^{-i\phi}}{2i}$, and $\cos \phi = \frac{e^{i\phi} + e^{-i\phi}}{2}$

may be of use.

3. Are L_x , L_y , L_z or L^2 eigenfunctions of $\psi(x)$ for the 3D potential well?

4. Determine $\langle r \rangle$, $\langle r^2 \rangle$, and σ_r for ψ_{100} . The integral identity $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ is helpful.

5. Determine $\langle p_r \rangle$, $\langle p_r^2 \rangle$, and σ_p for ψ_{100} . Note $(p_r)_{op} = -i\hbar \frac{\partial}{\partial r}$ and
- $$(p_r^2)_{op} = -\hbar^2 \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) = -\hbar^2 \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right).$$
6. If electrons were spinning balls, they would have moment of inertia $I = \frac{2}{5}MR^2$ and spin angular momentum $S = I\omega$. The maximum speed at which the outside edge of a real ball electron could be spinning is the speed of light. Since we know $|\vec{S}| = \sqrt{s(s+1)}\hbar = \sqrt{\frac{3}{4}}\hbar$, what value of R does this imply? Note that experiment has shown that electrons act like point particles down to distances below 10^{-15} m.
7. The deuteron is a hydrogen atom with an extra neutron in the core. The effects the reduced mass μ in our equation for E_n . If $m_p = 1836 m_e$ and $m_n = 1839 m_e$, find the wavelengths of light necessary to ionize a hydrogen atom and a deuteron. What is the difference in these two wavelengths?
8. We can define the effective charge that an electron sees by the formula $E_n = Z_{eff}^2 \frac{E_0}{n^2}$ where $E_0 = 13.6$ eV. The electron configuration of sodium is $1s^2 2s^2 2p^6 3s$. If the inner electrons fully shielded the valence electron from the core, what would Z_{eff} be? How much energy would be needed to remove the valence electron from sodium in this case? The actual ionization energy of sodium is 5.1 eV. What is the true Z_{eff} ?
9. Potassium has an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^6 4s$ and an ionization energy of 4.3 eV. What is the true Z_{eff} ? See previous question.
10. Write the electron configurations for the following, use only Fig. 7-19 of the text which shows the relative energies of the atomic shells and subshells.
- Rubidium, $Z = 37$.
 - Iodine, $Z = 53$.