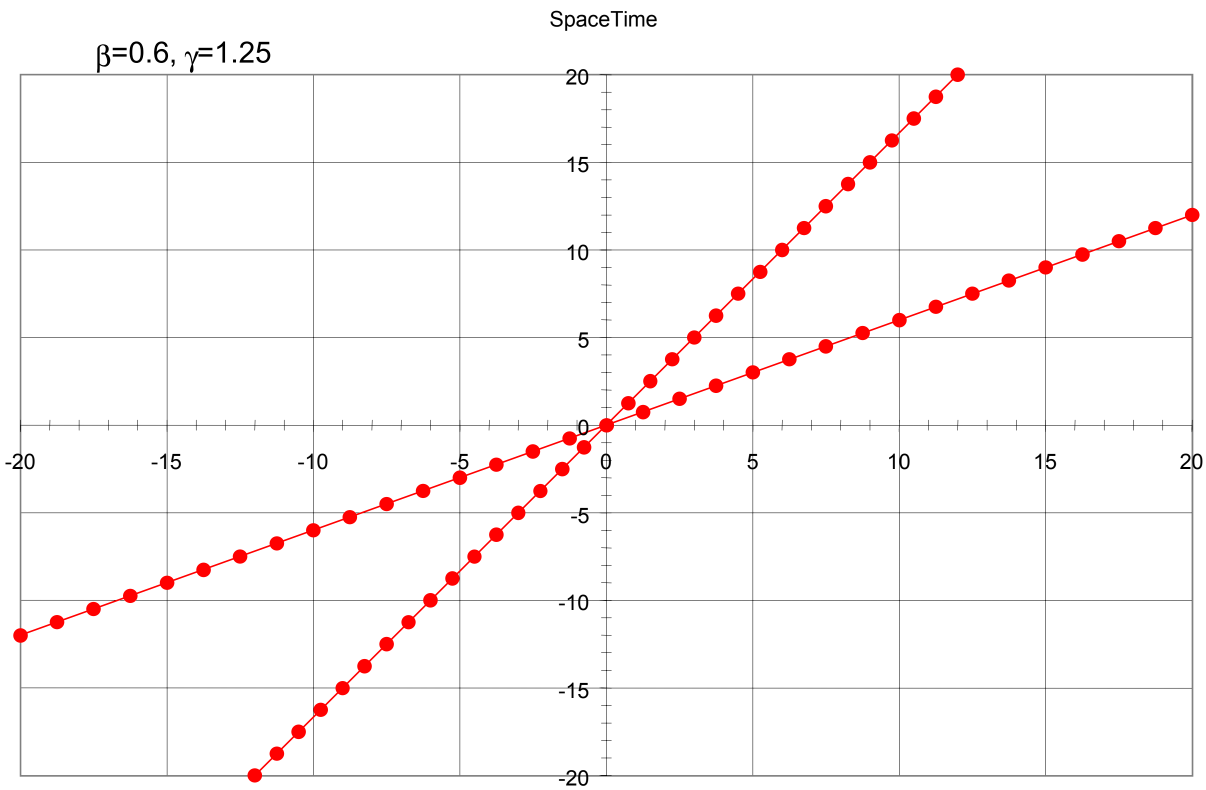
Spacetime Diagrams

Construction

A spacetime diagram has two parts. First there an ordinary rectangular ct vs x diagram for a stationary observer. This frame is called S. Superimposed on this diagram is the ct' vs x' axis for an observer moving at constant velocity $v = \beta c$ with respect to the stationary observer – frame S′. Because of the nature of spacetime as calculated from the Lorentz Transformation, this grid does not appear rectangular from the point of view of the stationary observer. The x′-axis will have slope β and the ct′-axis with have slope $1/\beta$. In both the speed of light is a line of slope 1.

Note that intervals on the x′ and ct′ axes are not the same size. On the x′-axis the intervals are $\Delta x' = \gamma \Delta x$. On the ct′-axis the intervals are $c\Delta t' = \gamma c \Delta t$.

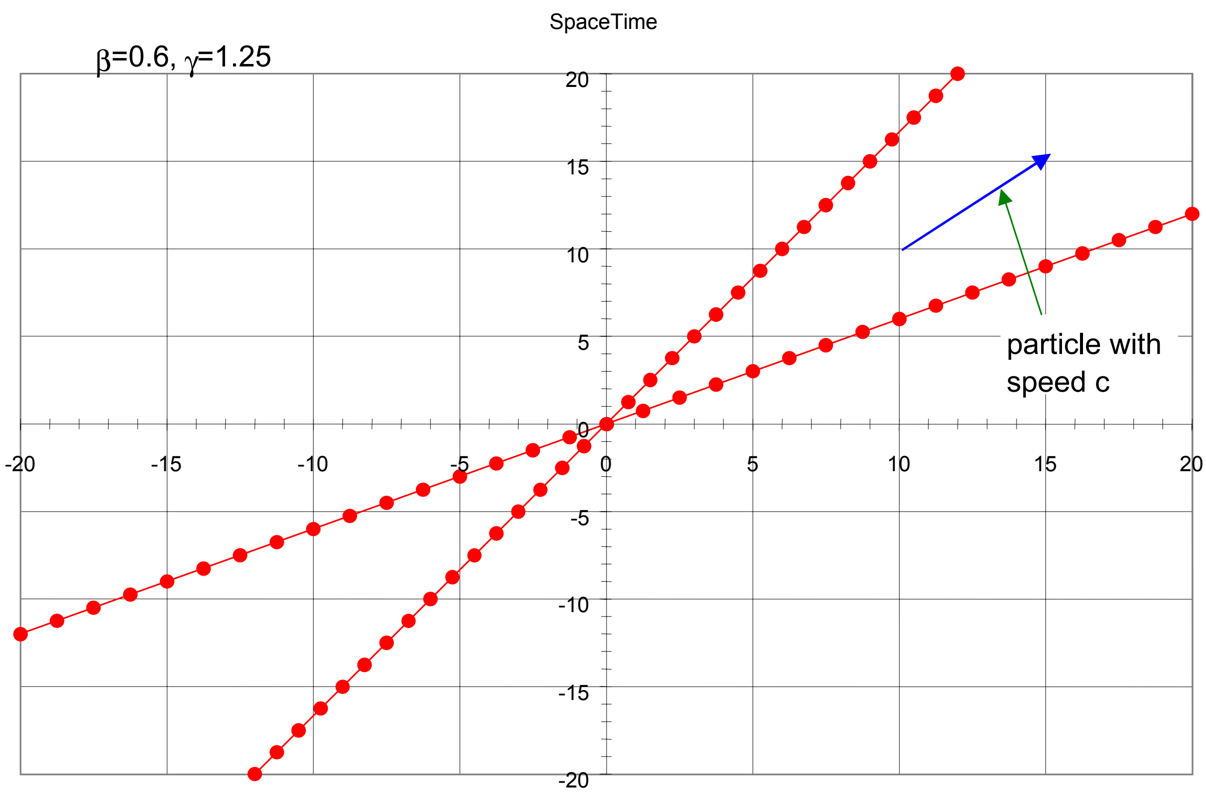
The spacetime diagram below has been drawn for $\beta = 0.6$ and $\gamma = 1.25$. The large red dots mark off intervals of one unit (the units could be metres , seconds, or whatever).



Worldline

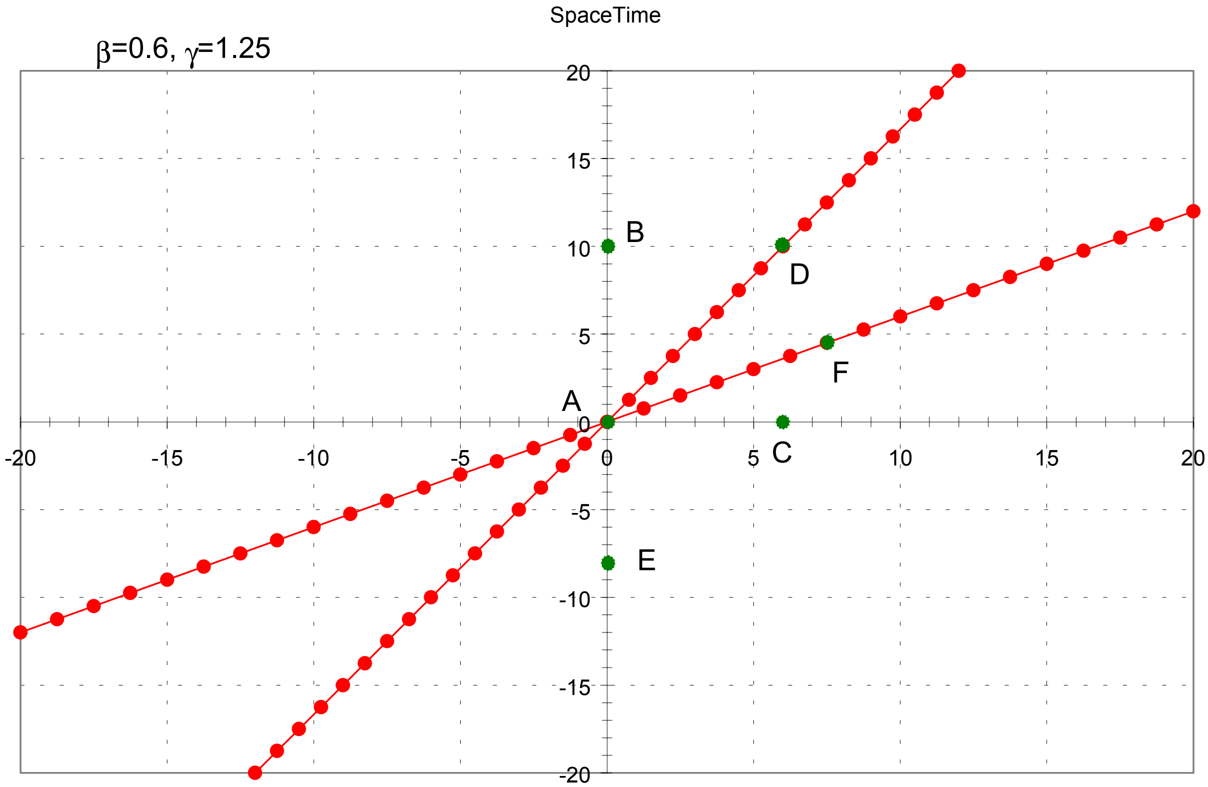
A worldline is the path of a particle on diagram. The worldline or path is exactly the same in both frames. The only difference is how each frame labels each point along the path. One uses (x, ct), the other (x′, ct′). These points are related by the Lorentz Transformation.

Note that the blue arrow in the diagram below. It is the worldline line for a particle moving to the right at the speed of light. Observers in both frames see the same speed u for it, where speed is u = 1/slope. Here u = 1/1 = 1 in both frames.

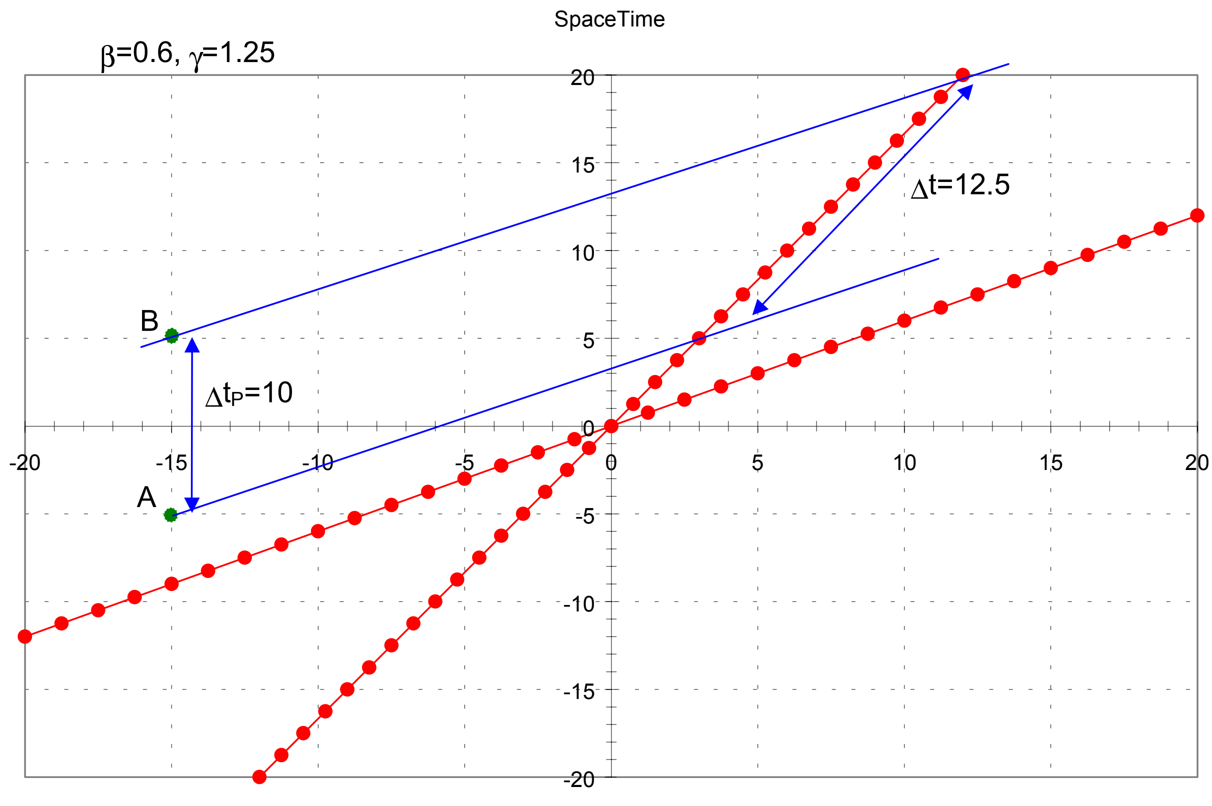


Proper Time

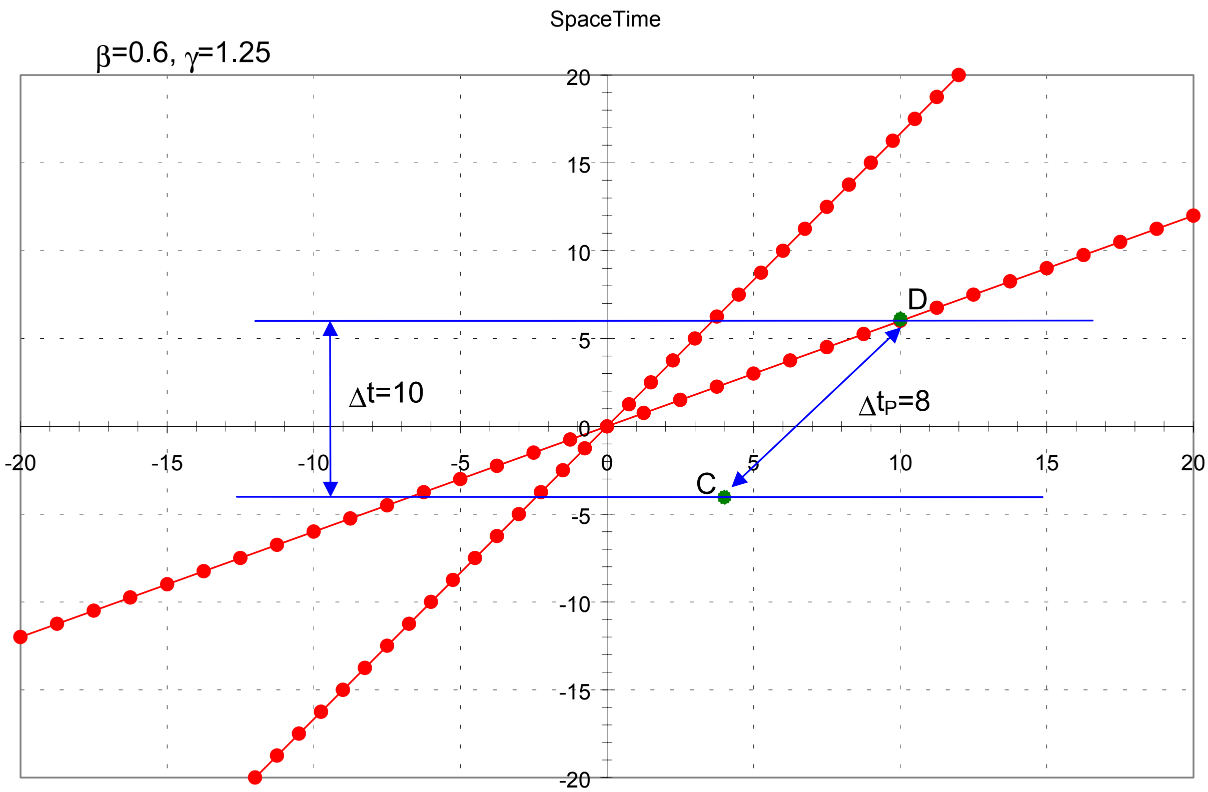
Proper time is time measured between two events by a clock at both events. The time interval in the other frame is related to the proper time by $\Delta t = \gamma\Delta t_P$. Six events, indicated by large green dots, and labelled A to F are shown on the graph. In S the time between A and B, Δt_{AB}, is a proper time because both are at x = 0. Similarly Δt_{CD} is a proper time in S. In S′ the time between A and D, $\Delta t_{AD}'$, is a proper time because both events occur at x′ = 0. Less obviously, $\Delta t_{EF}'$ is a proper time in S′ since both occur at x′ = 6.



The time interval in the other frame is related to the proper time by $\Delta t = \gamma\Delta t_P$ If we know Δt_P in one frame we calculate Δt in the other. This can be done graphically. Consider the interval between events A and B in the diagram below. Draw two lines parallel to the x′ axis through points A and B to intersect the ct′ axis. The interval between the intersection points is Δt the time interval in S′.



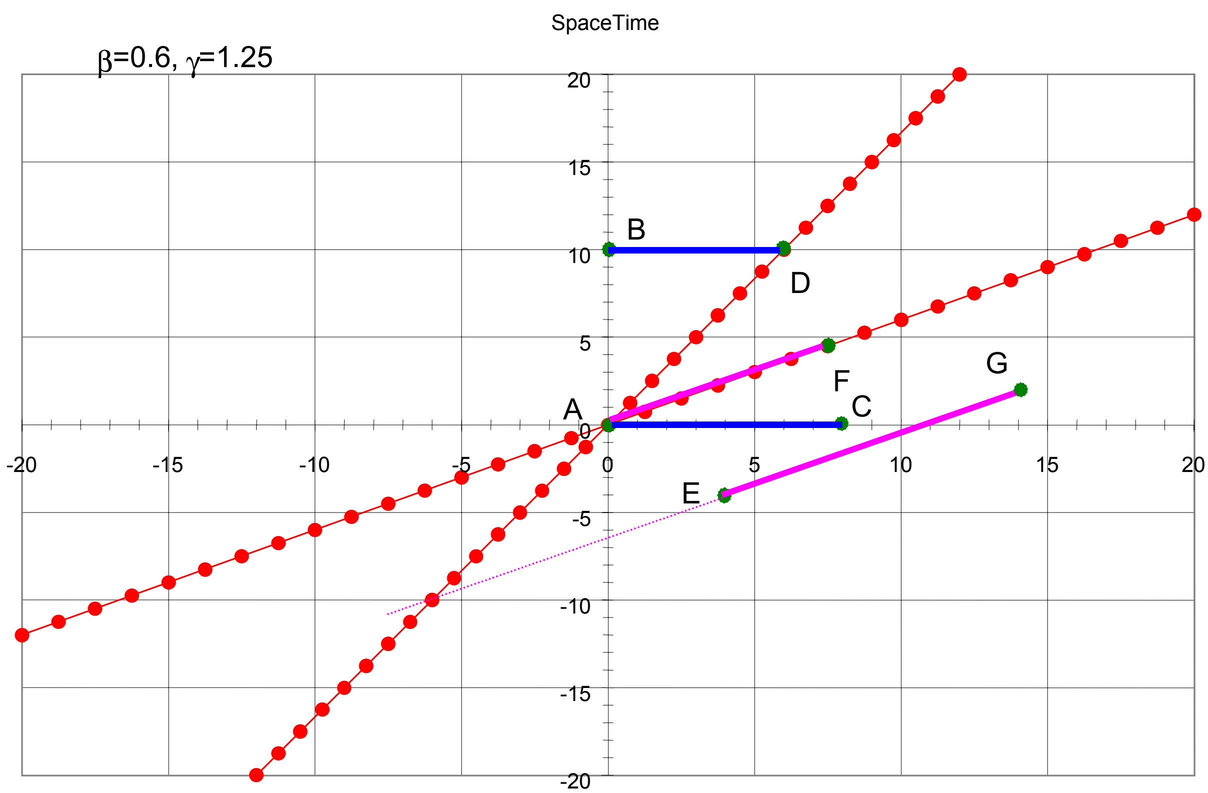
If the proper time is in the S′ frame, we need to draw lines that are parallel to the x axis (i.e. horizontal) through the points to intersect the t axis. This is shown in the diagram for the points C and D which are located at (8, 8) and (8, 0) in S′. The proper time interval is 8. The time interval in S is 10 as shown.



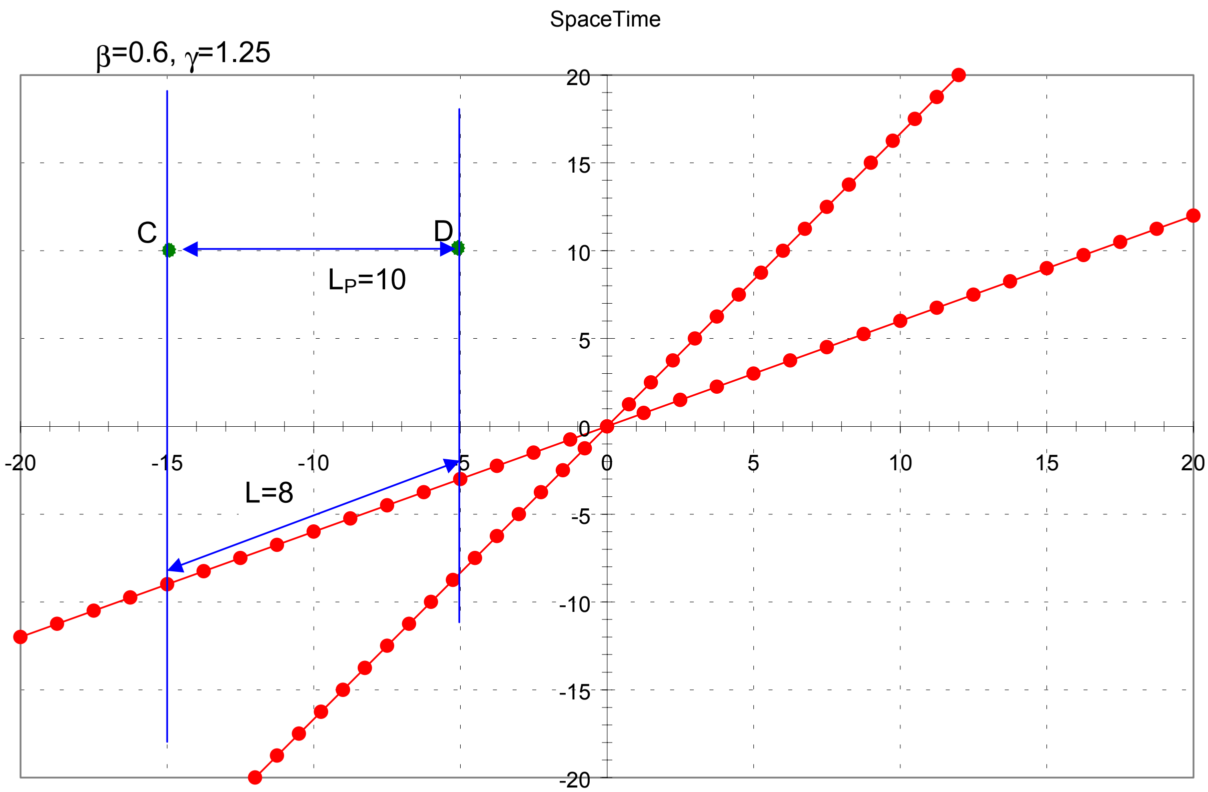
SpaceTime
β=0.6, γ=1.25
20
15
10
5
0
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
15
20
Δt=10
ΔtP=8
D
C

Proper Length

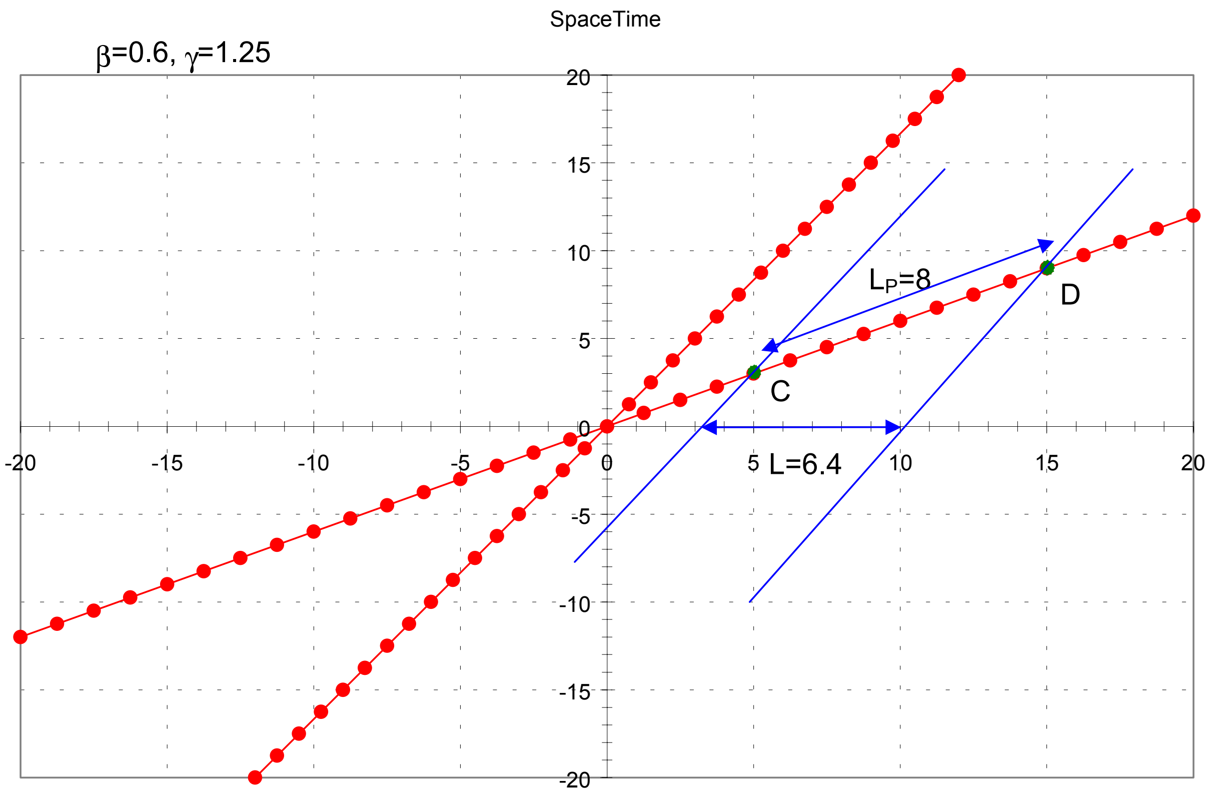
The proper length of an object is the length measured in a frame where the object is at rest. If an object is moving, you can only measure its length if you measure both ends at the same time. Six events, indicated by large green dots, and labelled A to G are shown on the graph below. In S the length between A and C, L_{AC}, is a proper length because both ends are measured at t = 0. Similarly L_{BD} is a proper time in S. In S′ the distance between A and D, L_{AD}', is a proper length because both ends occur at ct′ = 0. Less obviously, L_{EG}' is a proper length in S′ since both occur at x′ = -8.



Graphically if we know the proper length L_P in S, we can find the length in S′. The diagram below shows events C and D, the ends of some object. Of course in S, L_P can be measured at any time since it is not moving. The vertical blue lines show the other possible times. The length in S′ can only be determined by measuring the ends at the same time in S′. The intersection of the blue lines with the x′ axis, ct′ = 0, yields the length in S′.



Similarly if we know a proper length in S′ we can determine the length in S from the graph. The diagram below shows events C and D, the ends of some object. In S′, L_P can be measured at any time since it is not moving. The titled blue lines, which are parallel to the ct′ axis, show the other possible times. The length in S can only be determined by measuring the ends at the same time in S′. The intersection of the blue lines with the x axis, ct = 0, yields the length in S.



SpaceTime
β=0.6, γ=1.25
L_P=8
L=6.4
C
D
20
15
10
5
0
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
15
20