Solving SR problems

Point #1 You use simple kinematics to solve most problems $v = d/t$ and $d = vt$. Care must be exercised in using distance and time given in problems. Both must belong to the same frame. People travelling relative to one another see different distances and times because of Length Contraction and Time Dilation.

Example A star is 10 c-y away from earth and a spaceship pilot leaves earth and finds it takes him 7 y to get there. What is the spaceship's speed?

10 c-y is seen by someone on earth, the spaceship pilot does not see 10 c-y. 7 y is what the pilot records not what someone on earth measures.

Using both together gives a nonsense result

$$v_{silly} = 10 \text{ c-y} / 7 \text{ y} = 1.43 \text{ c}$$

which is impossible as you cannot travel faster than the speed of light.

So how do we find the speed? We make use of the Length Contraction and Time Dilation relationships. That requires that you correctly identify the proper time in the problem and the proper distance – if there are any given and there may not be. Here earth measures a proper distance because the earth and star are not moving with respect to one another (at least not at relativistic speeds). Also the time measured by the pilot is a proper time since he is there when he leaves earth and when he arrives at the star.

So in the pilot's frame, the star is closer because of Length Contraction at (10 c-y)/γ away where we don't know γ yet. His speed is thus

$$v_{pilot} = (10 \text{ c-y}/\gamma)/7\text{y} = (10/7\gamma) \text{ c}$$

or

$$\gamma\beta_{pilot} = 10/7$$

where $\beta_{pilot} = v_{pilot}/c$.

This is

$$\beta_{pilot}/[1 - \beta_{pilot}^2]^{½} = 10/7 .$$

Solving yields $\beta_{pilot} = 0.819$ and $\gamma = 1.7438$. The distance to the star is thus 5.7346 c-y in the pilot's frame.

In the earth frame, the trip takes longer because than the pilot observes because of Time Dilation, at $\gamma(7\ \text{y})$ where again we wouldn't know γ. To a person on earth the speed of the ship is observed to be

$$v_{observed} = (10\ \text{c-y}) / \gamma(7\ \text{y}) = 10/7\gamma\ c$$

just as before. The person on earth sees the ship moving at 0.819c as well. The earth observer says the trip actually takes 12.2066 years.

Important point! Both must measure the same speed for each other since speed is relative.

Point #2 The spacetime interval is invariant. If you have done a problem correctly, all parties must see the same spacetime interval.

Example A star is 10 c-y away from earth and a spaceship pilot leaves earth and finds it takes him 7 y to get there. Compare the spacetime intervals.

The spacetime interval is defined $(\Delta S)^2 = (ct)^2 - (\Delta x)^2$. For the pilot ct = 7 c-y and Δx is 0 since both events occur at the same place as the pilot (i.e. the star came to the pilot). Thus

$$(\Delta S_{pilot})^2 = (7\ \text{c-y})^2 - (\ 0\ )^2 = (7\ \text{c-y})^2\ .$$

For the earth $ct = \gamma(7\ \text{c-y}) = 1.7438 \times (7\ \text{c-y}) = 12.2066$ c-y and $\Delta x = 10$ c-y. Hence

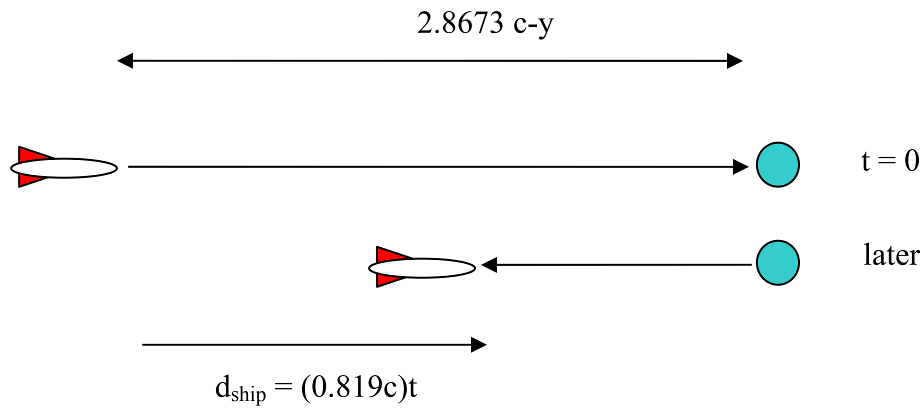
$$(\Delta S_{earth})^2 = (12.2066\ \text{c-y})^2 - (10\ \text{c-y})^2 = (7\ \text{c-y})^2\ .$$

These agree. So our previous answers appear correct or at least consistent.

Point #3 Light travels at a finite speed. You often need to take this travel time into account.

Example A star is 10 c-y away from earth and a spaceship pilot leaves earth and finds it takes him 7 y to get there. Halfway to the star he sends a signal to the base at the star which immediately returns the signal. Find out when the pilot receives the signal in both frames.

In the frame of the pilot, halfway is $(10\ \text{c-y})/\gamma/2 = 2.8673$ c-y. So obviously it takes 2.8673 years for the signal to reach the base which then replies. During this time the ship and signal are both moving. Let's draw a little sketch. Sketches are always a good idea!



Considering the sketch we can see the simple relation that

$$ct + (0.819c)t = 2 \times (2.8673 \text{ c-y})$$

or t = 3.1526 years. This is a proper time so earth would see $\gamma \times 3.1526$ y = 5.4975 years.