SR Kinematics

Consider any two observers that are moving with respect to one another.

What they agree about	What they disagree about
relative speed of each other	length or distance
speed of light	time and time intervals
laws of physics including v = d/t	simultaneity

Identify which measurement belongs to which observer. Do not mix frames! Calculations are done in one frame and the Time Dilation, Length Contraction, or the Lorentz Transform formulas are used to get values in other frames.

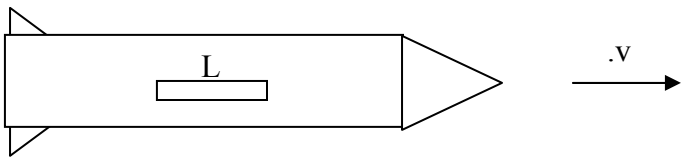
Identify which observer, if any, measures a proper length. For a length to be proper, and if the object is moving, both the start and endpoint of the object must be measured at the same time. If the object is not moving in the observer's frame, then it's length is proper. All other observers see a shorter length than the proper length by $L_v = \gamma(v)L_{proper}$, where L_v is the length the other observer sees and v is the relative speed of the observer with respect to the first observer. Length contraction only happens in the direction of motion.

Identify which observer, if any, measures a proper time interval. For a time interval to be proper, both the observer and his watch must be at the located at the starting point and at the endpoint of the interval. All other observers see a longer time interval than the proper time by $t_v = t_{proper}/\gamma(v)$, where t_v is the time interval the other observer sees and v is the relative speed of the observer with respect to the first observer.

There does not need to be a proper length or proper time interval in a problem. In such cases the Lorentz Transform must be used.

Example

A planet is 12 c-years distant from earth. A spaceship leaves earth and travels to that planet at 0.6 c. The pilot of the spaceship says the trip takes 16 years. During the trip the bored pilot measures the length of his spaceship and finds it to be 200 m long. He measures its width to be 25 m. He has a transparent block that is L = 3 m long. He put a light detector/clock/mirror combination at both ends of the block. He shines a laser beam through the block and bounces it off the detector at the far end so that it returns to the detector at the near end. He finds that the halfway time for the light beam from the reading on the clocks at both ends to be 0.200 ns and the roundtrip time to be 4.00 ns. From this he determines the speed of light in the material to be 0.5 c.



(a) In which frame is the 12 c-years measured?
(b) Is the 12 c-years a proper time? Why or why not?
(c) Will the pilot and earth agree on this distance? If not, who if anyone measures the bigger distance?
(d) In whose frame is the relative speed 0.6 c?
(e) Will the pilot and earth agree on this speed? If not, who if anyone measures the bigger speed?
(f) In which frame is the 16 years measured?
(g) Is the 16 years a proper time? Why or why not?
(h) Will the pilot and earth agree on this time? If not, who if anyone measures the bigger time interval?
(i) In which frame is the 200 m spaceship measured?
(j) Is the 200 m a proper distance? Why or why not?
(k) Will the pilot and earth agree on this distance? If not, who if anyone measures the bigger distance?
(l) In which frame is the 25 m width spaceship measured?
(m) Is the 25 m a proper distance? Why or why not?
(n) Will the pilot and earth agree on this distance? If not, who if anyone measures the bigger distance?
(o) In which frame is the 3 m transparent block measured?
(p) Is the 3 m a proper distance? Why or why not?
(q) Will the pilot and earth agree on this distance? If not, who if anyone measures the bigger distance?
(r) In which frame is the 0.200 ns half trip time measured?
(s) Is the 0.200 ns a proper time? Why or why not?
(t) Will the pilot and earth agree on this time? If not, who if anyone measures the bigger time interval?
(u) In which frame is the 0.400 ns round trip time measured?
(v) Is the 0.400 ns a proper time? Why or why not?
(w) Will the pilot and earth agree on this time? If not, who if anyone measures the bigger time interval?
(x) In which frame is the speed of light in the block, 0.5 c, measured?
(y) Will the pilot and earth agree on this speed? If not, who if anyone measures the bigger speed?

Answers:

(a) Earth.
(b) Technically no since both earth and planet and their respective suns are moving. In practice, it is a proper distance since these speeds are very small compared to the speed of light.
(c) No, they will not agree. The earth sees the bigger distance since it measures the proper distance. All others must see a shorter distance.
(d) Both. Speed is relative.
(e) Yes, speed is relative.
(f) Pilot.
(g) Yes, the pilot and his watch are there when the trip starts and ends.
(h) No, the earth sees a longer time since the pilot measured the proper time.
(i) Pilot.
(j) Yes the ship is not moving with respect to the pilot.
(k) No, they will not agree. The pilot measures the biggest length; all others must see a shorter distance.
(l) Pilot.
(m) Yes, the ship is not moving with respect to the pilot.
(n) Yes, lengths perpendicular to the direction of travel are not affected by length contraction.
(o) Pilot.
(p) Yes, the block is not moving with respect to the pilot.
(q) No, they will not agree. The pilot measures the biggest length; all others must see a shorter distance.
(r) Pilot.
(s) No, pilot used two clocks/detectors at different locations.
(t) No, they will not agree. This earth observer will measure a longer time interval. To see this note that a third observer travelling at the speed of the light in the material, i.e. 0.5 c with respect to the pilot, would be there when the light enters the block and when it leaves and so would measure the proper time. The pilot would see a longer time interval $t_{pilot} = t_{proper}/\gamma(0.5\ c)$, but the earth would measure and even longer time interval $t_{pilot} = t_{proper}/\gamma(v')$ since the relative velocity v' with respect to earth is greater than 0.5 c.
(u) Pilot.
(v) Yes, the near clock/detector measures both events; the light entering the block and returning.
(w) No. Earth would measure the longer time interval.
(x) Pilot.
(y) No. Earth would measure the bigger speed since the pilot measure .5 c forward and he is moving at .6 c with respect to earth.