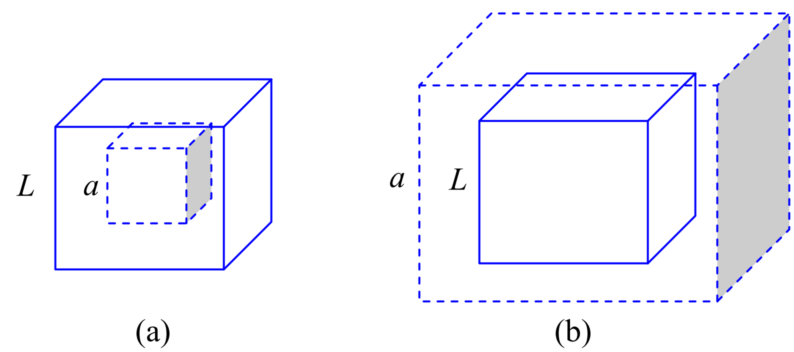
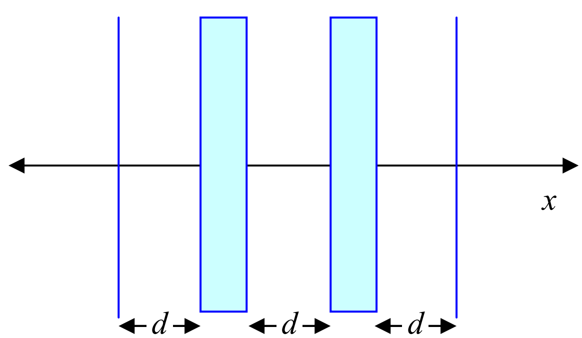
1. A student encounters a problem that has a cube of side length L and constant charge density ρ. The problem asks for the electric field everywhere along a line coming from the centre cube and passing through the centre of a face. A sketch must also be drawn. The problems seem familiar so the students says this must be a Gauss' Law problem.
   (a) The object is symmetric with uniform density and thus the field is symmetric too.
   (b) The obvious Gaussian surface is a cube of side length a sharing the same centre as the cube.
   (c) There are two regions as shown below.



   (d) Because of the six sides, he concludes that the flux $\phi = \int_{surface} \mathbf{E} \bullet \boldsymbol{n} dA = 6Ea^2$.
   (e) In diagram (a) Q_{inside} is just ρa^3, so Gauss' Law gives $6Ea^2 = \rho a^3/\varepsilon_0$ or $E = \rho a/6\varepsilon_0$.
   (f) In diagram (b) Q_{inside} is just ρL^3, so Gauss' Law gives $6Ea^2 = \rho L^3/\varepsilon_0$ or $E = \rho L^3/6a^2\varepsilon_0$.

   The student is quite pleased with his cleverness, especially as the electric field drops as a^2 as it must if you are far away from a charge so that it looks like a point charge. Unfortunately he is mistaken. Explain exactly where he went wrong.

2. The diagram below shows the side view of large sheets of charge. The outer sheets are extremely thin and carry identical surface charge σ. The inner sheets are thicker and have identical constant uniform volume charge density ρ. The inner sheets are distance d apart. The outer sheets are d from the inner plates. The inner plates are each ½d thick.
   (a) Find the electric field everywhere along the x axis stopping when you are beyond but still "close" to the outside sheet.
   (b) Sketch the field.
   (c) What does σ have to be in terms of ρ for there to be no field beyond the outer sheets?
   (d) Explain why one needs to stop while still "close" to the outside sheet (i.e. stop using Gauss' Law).



3. Repeat the previous question, parts (a) and (b), if the charge densities are σ, -ρ, ρ, and -σ as you go from left to right.

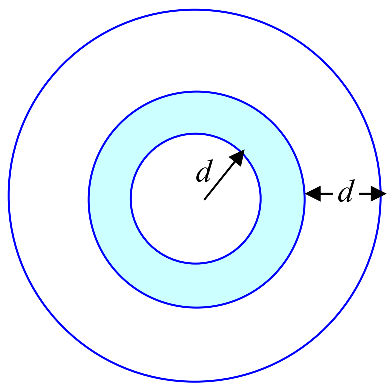
4. The diagram below shows the end view of large cylindrical sheets of charge. The outer sheet is extremely thin and carries identical surface charge σ. The inner sheet is thicker and has identical constant uniform volume charge density ρ. The inside of the thick cylinder has radius *d*. The outer thin cylindrical sheet is *d* from the outside of the inner cylinder. The inner cylinder is ½*d* thick.

(a) Find the electric field everywhere along the radial axis stopping when you are beyond but still "close" to the outside cylindrical sheet and not too close to the ends of the cylinder.

(b) Sketch the field.

(c) What does σ have to be in terms of ρ for there to be no field beyond the outer cylinder?

(d) Explain why one needs to stop while still "close" (i.e. stop using Gauss' Law).



5. The diagram above (doing double duty) shows the cross-section view of spherical shells of charge. The outer shell is extremely thin and carries identical surface charge σ. The inner shell is thicker and has identical constant uniform volume charge density ρ. The inside of the thick spherical shell has a radius *d*. The outer thin spherical shell is *d* from the outside of the inner shell. The inner shell is ½*d* thick.

(a) Find the electric field everywhere along the radial axis. Sketch the field.

(b) What does σ have to be in terms of ρ for there to be no field beyond the outer shell?

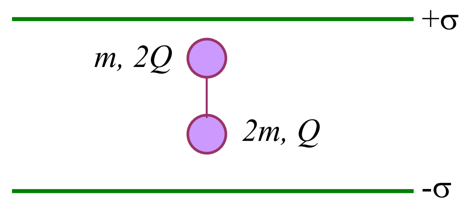
(c) Explain why one <u>does not</u> need to stop while still "close" to the shell.

6. Two small balls connected by a small thread are floating in the air between the vertically separated plates of a parallel plate capacitor as shown in the diagram below. The plates have surface charge density σ as shown. The mass and charge of each ball are also shown.

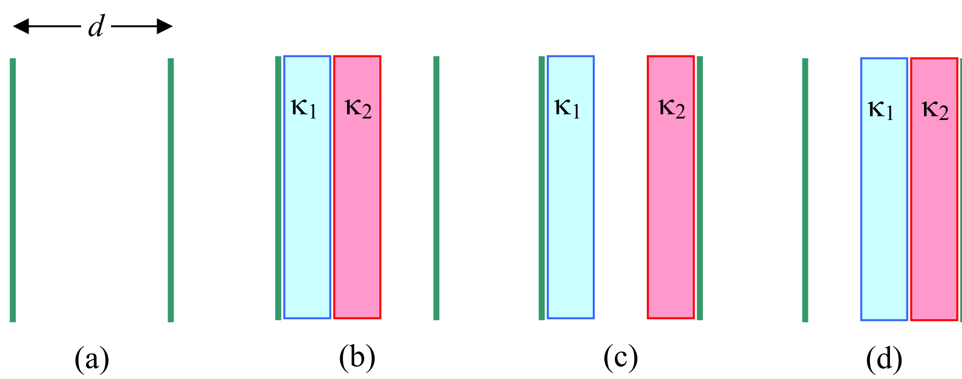
(a) What is the sign of the charge Q? Explain.

(b) Obtain an expression for Q.

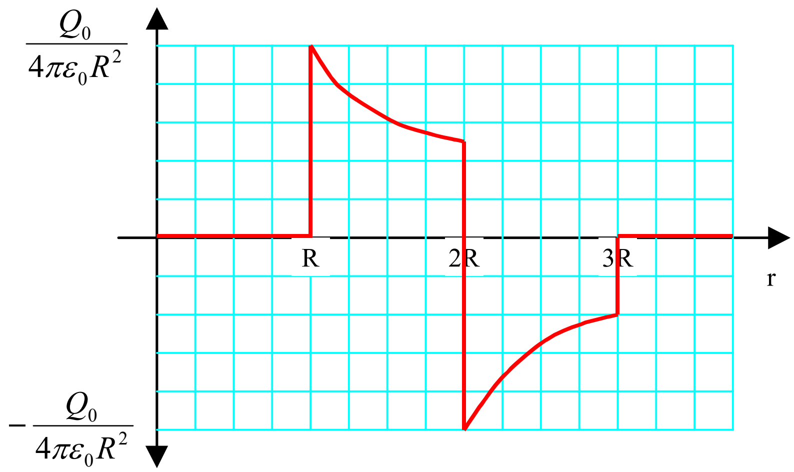
(c) If the charge on the bottom ball had been $-Q$, what would Q have been?



7. A parallel plate capacitor has plate area *A* and separation *d* as shown below in diagram (a) below. Insulators of dielectric constants κ_1 and κ_2 and equal width $^1/_3 d$ are placed between the plates in the configurations shown in (b), (c), and (d).
   - (i) Find the capacitance in each case. (Try to do this in two different ways!)
   - (ii) The bare capacitor in (a) is connected to a battery of voltage V_0. While still connected the insulators are inserted. Find the surface charge density on the plates and the insulator surfaces in each case.
   - (iii) The bare capacitor in (a) is connected to a battery of voltage V_0 and fully charged. Then the battery is disconnected and the insulators are inserted. Find the surface charge density on the plates and the insulator surfaces in each case.

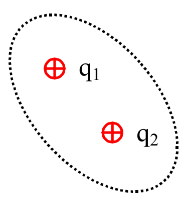


8. The diagram below is a sketch of the electric field due to a set of concentric thin-shelled (i.e. no thickness) spherical shells as a function of the distance from their common centre.
   (a) How many shells are there in total? Explain how you know.
   (b) What is the charge on each shell in multiples of Q_0? Again explain your reasoning.
   Hint, remember that the surface area of a sphere is $4\pi r^2$ where r is the radius of the sphere.

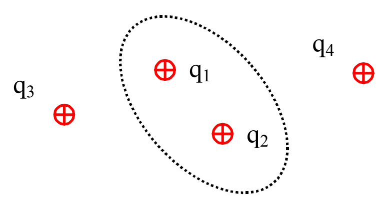


9. In diagrams (i) and (ii) below, the dashed line denotes a Gaussian surface. In diagram (i) there are two charges q_1 and q_2. In diagram (ii), two extra charges, q_3 and q_4, have been added with no other changes. Is the net flux ϕ through the Gaussian surface

(a) greater in (i) than in (ii),
(b) equal in (i) and (ii), or
(c) less in (i) than in (ii).
Explain.



(i)



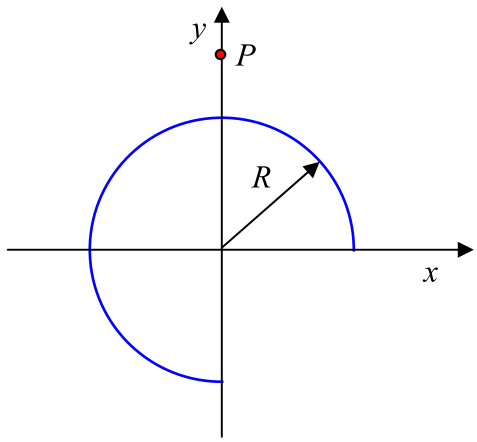
(ii)

10. A spherical distribution of charge produces the radial electric field

$$E(r) = \begin{cases} 0 \quad - & 0 < r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & R < r < 2R \\ -\dfrac{Q_1}{4\pi\varepsilon_0 r^2} & r > 2R \end{cases}$$

(a) Find the electrostatic potential difference between a point *a* lying between 0 and *R* and a point *b* lying beyond *2R*.
(b) Find the Q_1 in terms of Q such that the electrostatic potential difference between a point at zero and a point at infinity is zero.

11. The curved object below has a charge Q uniformly distributed along its length. The radius of curvature is R. Derive expressions for the components of the electric field at a point P a distance a from the centre along the y axis. Hint: find $\mathbf{r}$ in terms of θ. Do not use the Law of Cosines.



12. The curved object below has a charge density λ uniformly distributed along its length. The radius of curvature is R. Derive an expressions for the electric potential at a point P at the centre of the missing arc. Hint: find r in terms of θ. You may use the Law of Cosines.