PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(Section 15.1)

Linear momentum: $\boldsymbol{L} = m\boldsymbol{v}$

- vector m***v*** is called the linear momentum
- denoted as ***L*** (***P*** in 1120)
- vector has the same direction as ***v***.
- units of (kg·m)/s or (slug·ft)/s.

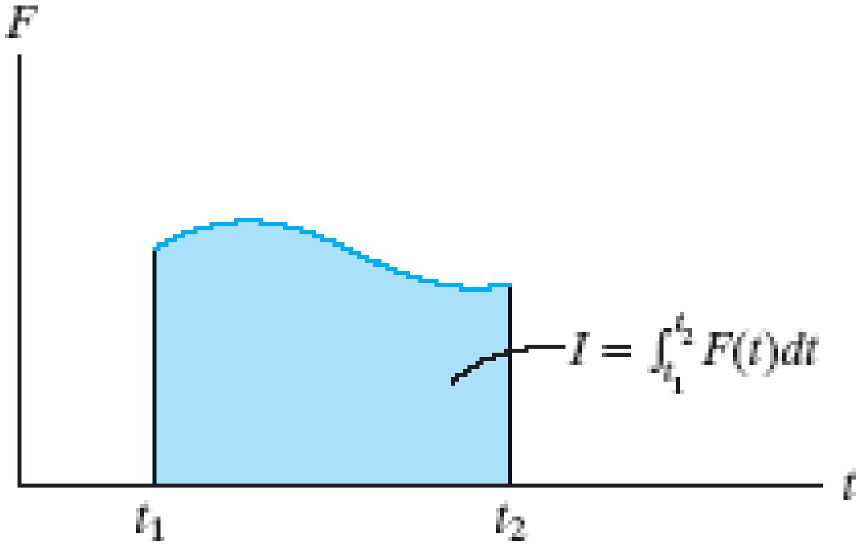
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM

(Section 15.1)

Linear impulse: $\mathbf{I} = \int \mathbf{F}\, dt$

- is the linear impulse, denoted ***I***.
- is a vector quantity measuring the effect of a force during its time interval of action.
- ***I*** acts in the same direction as ***F***
- has units of N·s or lb·s.

The impulse ***I*** may be determined by direct integration. Graphically, *it* can be represented by the area under the force versus time curve.



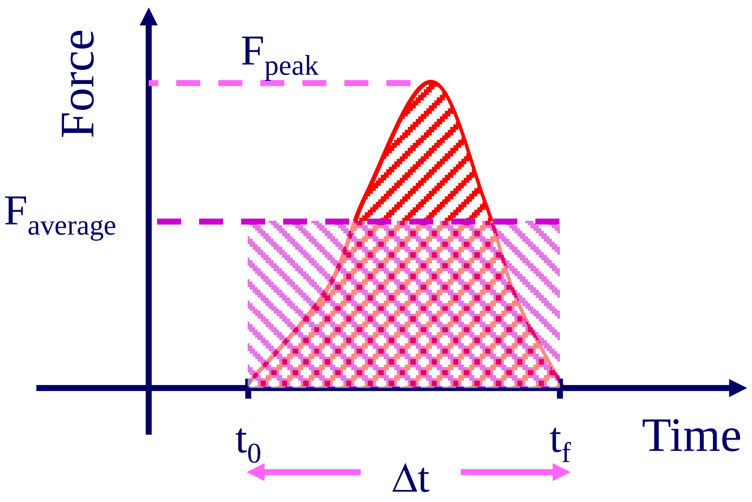
Variable Force

If ***F*** is constant, then

$$\boldsymbol{I} = \boldsymbol{F}\,(t_2 - t_1)\,.$$

The average force is defined by requiring it to give the same Linear Impulse

$$\boldsymbol{F_{avg}} = \boldsymbol{I} \,/\, (t_2 - t_1)\,.$$



PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM
(continued)

The relationship between Impulse and momentum next is obtained by integrating the equation of motion with respect to time.

The result is referred to as the principle of impulse and momentum. It can be applied to problems involving both linear and angular motion.

This principle is useful for solving problems that involve force, velocity, and time. It can also be used to analyze the mechanics of impact (taken up in a later section).

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM (continued)

The equation of motion can be written

$$\sum \boldsymbol{F} = m\,\boldsymbol{a} = m\,(d\boldsymbol{v}/dt)$$

Separating variables and integrating between the limits $\boldsymbol{v} = \boldsymbol{v}_1$ at $t = t_1$ and $\boldsymbol{v} = \boldsymbol{v}_2$ at $t = t_2$ results in

$$\sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m \int_{\boldsymbol{v}_1}^{\boldsymbol{v}_2} d\boldsymbol{v} = m\boldsymbol{v}_2 - m\boldsymbol{v}_1$$

$$m\boldsymbol{v}_1 + \sum \int_{t_1}^{t_2} \boldsymbol{F}\,dt = m\boldsymbol{v}_2$$

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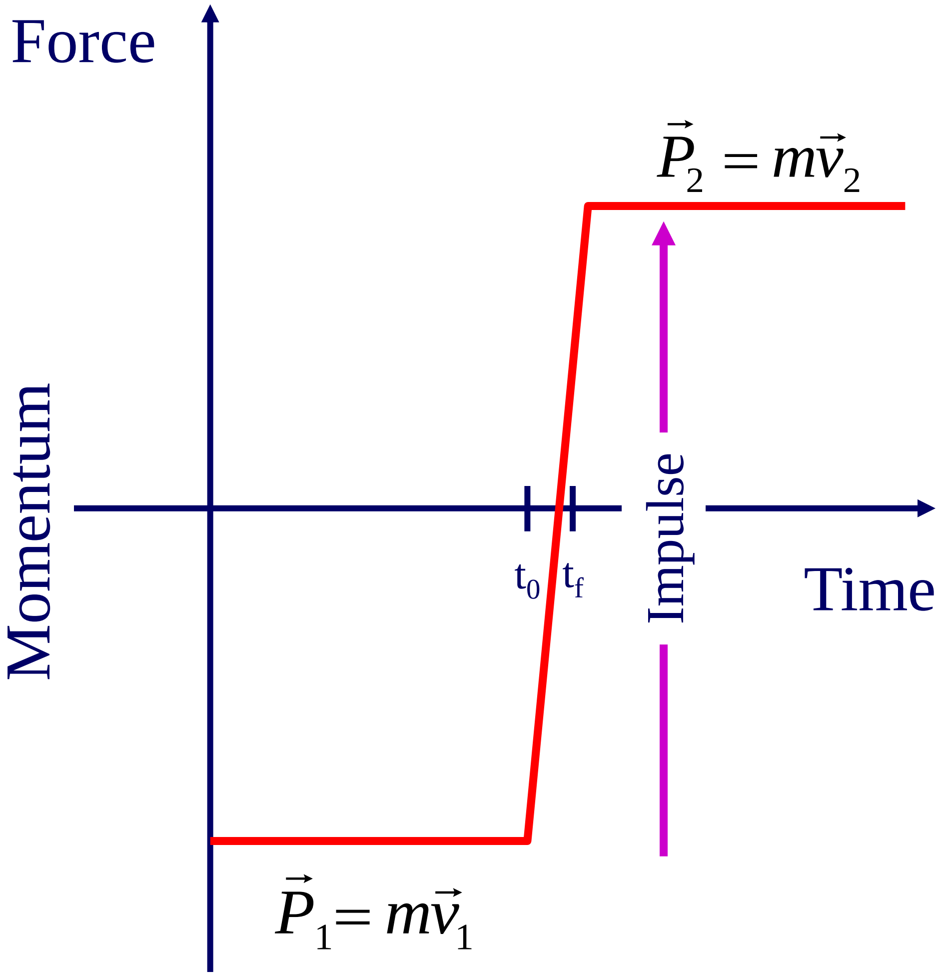


IMPULSE AND MOMENTUM: SCALAR EQUATIONS

Since the principle of linear impulse and momentum is a vector equation, it can be resolved into its x, y, z component scalar equations:

$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$

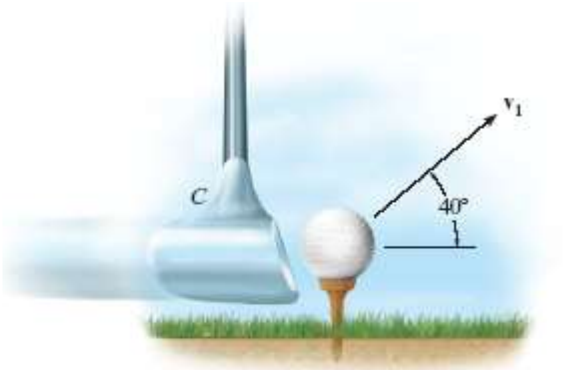
$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

The scalar equations provide a convenient means for applying the principle of linear impulse and momentum once the velocity and force vectors have been resolved into x, y, z components.

PROBLEM SOLVING

- Establish the x, y, z coordinate system.
- Draw the particle's free body diagram and establish the direction of the particle's initial and final velocities, drawing the impulse and momentum diagrams for the particle. Show the linear momenta and force impulse vectors.
- Resolve the force and velocity (or impulse and momentum) vectors into their x, y, z components, and apply the principle of linear impulse and momentum using its scalar form.
- Forces as functions of time must be integrated to obtain impulses. If a force is constant, its impulse is the product of the force's magnitude and time interval over which it acts.

EXAMPLE



Given: A 40 g golf ball is hit over a time interval of 3 ms by a driver. The ball leaves with a velocity of 35 m/s, at an angle of 40°. Neglect the ball's weight while it is struck.

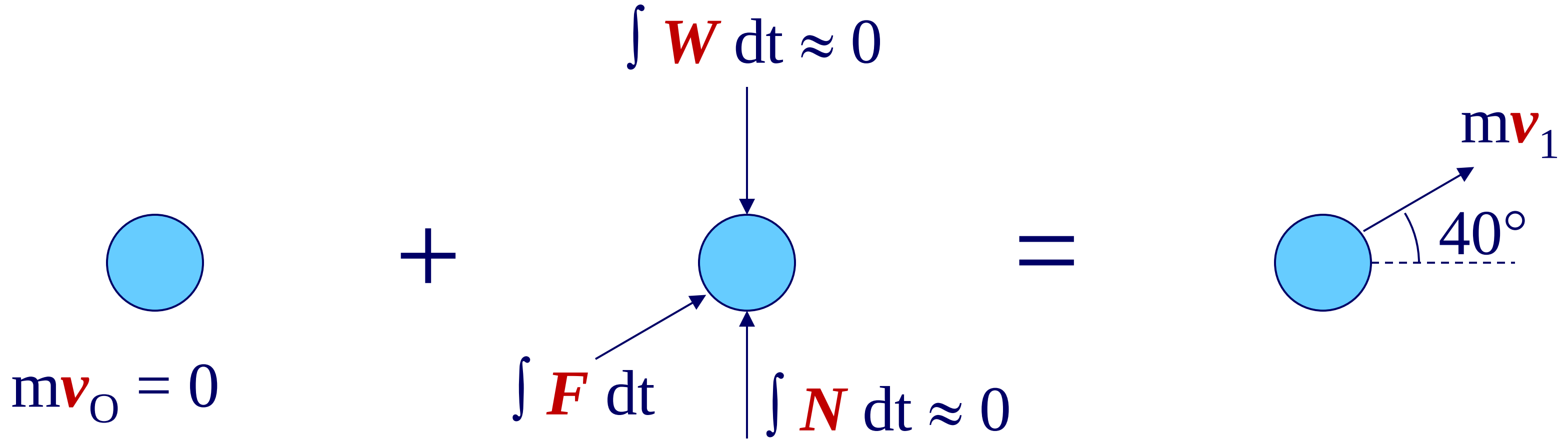
Find: The average impulsive force exerted on the ball and the momentum of the ball 1 s after it leaves the club face.

Plan:
1) Draw the momentum and impulsive diagrams of the ball as it is struck.
2) Apply the principle of impulse and momentum to determine the average impulsive force.
3) Use kinematic relations to determine the velocity of the ball after 1 s. Then calculate the linear momentum.

EXAMPLE
(continued)

Solution:

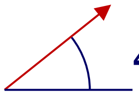
1) The impulse and momentum diagrams can be drawn:



The impulse caused by the ball's weight and the normal force $\boldsymbol{N}$ can be neglected because their magnitudes are very small as compared to the impulse of the club. Since the initial velocity ($\boldsymbol{v}_O$) is zero, the impulse from the driver must be in the direction of the final velocity ($\boldsymbol{v}_1$).

EXAMPLE
(continued)

2) The principle of impulse and momentum can be applied along the direction of motion:

 40°

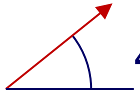
$$mv_O + \sum \int_{t_0}^{t_1} F\,dt = mv_1$$

The average impulsive force can be treated as a constant value over the duration of impact. Using $v_O = 0$,

$$0 + \int_{0}^{0.003} F_{avg}\,dt = mv_1$$

$$F_{avg}(0.003 - 0) = mv_1$$

$$(0.003)\,F_{avg} = (0.04)(35)$$

$F_{avg} = 467$ N  40°

EXAMPLE
(continued)

3) After impact, the ball acts as a projectile undergoing free-flight motion. Using the constant acceleration equations for projectile motion:

$$v_{2x} = v_{1x} = v_1 \cos 40° = 35 \cos 40° = 26.81 \text{ m/s}$$

$$v_{2y} = v_{1y} - gt = 35 \sin 40° - (9.81)(1) = 12.69 \text{ m/s}$$

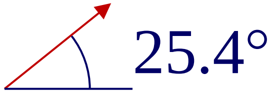
$$\Rightarrow \quad \mathbf{v}_2 = (26.81\, \mathbf{i} + 12.69\, \mathbf{j}) \text{ m/s}$$

The linear momentum is calculated as $\mathbf{L} = m\,\mathbf{v}$.

$$\mathbf{L}_2 = m\mathbf{v}_2 = (0.04)(26.81\, \mathbf{i} + 12.69\, \mathbf{j}) \text{ (kg·m)/s}$$

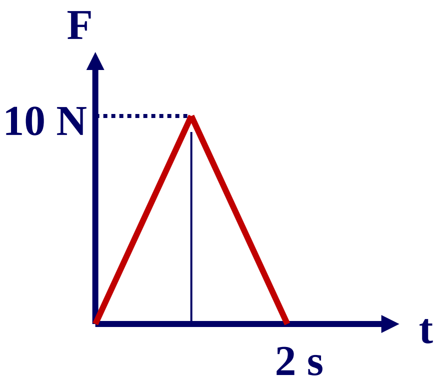
$$\mathbf{L}_2 = (1.07\, \mathbf{i} + 0.508\, \mathbf{j}) \text{ (kg·m)/s}$$

$$L_2 = 1.18 \text{ (kg·m)/s}$$



CONCEPT QUIZ

1. Calculate the impulse due to the force.



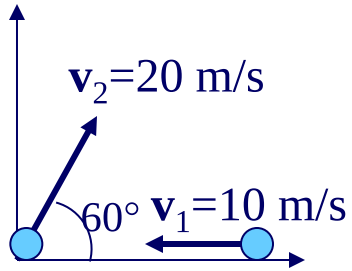
A) 20 kg·m/s B) 10 kg·m/s

C) 5 N·s D) 15 N·s

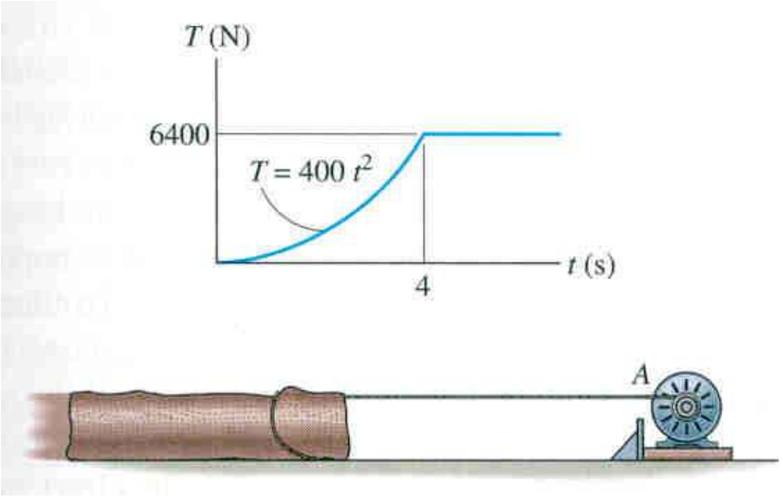
2. A constant force ***F*** is applied for 2 s to change the particle's velocity from $\mathbf{v}_1$ to $\mathbf{v}_2$. Determine the force ***F*** if the particle's mass is 2 kg.

A) (17.3 ***j***) N B) (–10 ***i*** +17.3 ***j***) N

C) (20 ***i*** +17.3 ***j***) N D) (10 ***i*** +17.3 ***j***) N



GROUP PROBLEM SOLVING



Given: The 500 kg log rests on the ground (coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$). The winch delivers a towing force ***T*** to its cable at A as shown.

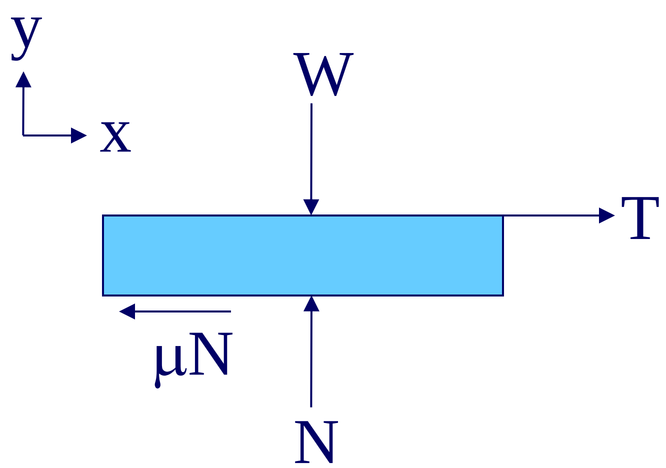
Find: The speed of the log when t = 5 s.

Plan:
1) Draw the FBD of the log.
2) Determine the force needed to begin moving the log, and the time to generate this force.
3) After the log starts moving, apply the principle of impulse and momentum to determine the speed of the log at t = 5 s.

GROUP PROBLEM SOLVING
(continued)

Solution:

1) Draw the FBD of the log:



$\sum F_y = 0$ leads to the result that $N = W = mg = (500)(9.81) = 4905$ N. Before the log starts moving, use μ_s. After the log is moving, use μ_k.

2) The log begins moving when the towing force T exceeds the friction force $\mu_s N$. Solve for the force, then the time.

$$T = \mu_s N = (0.5)(4905) = 2452.5 \text{ N}$$
$$T = 400\, t^2 = 2452.5 \text{ N}$$
$$t = 2.476 \text{ s}$$

Since $t < 4$ s, the log starts moving before the towing force reaches its maximum value.

GROUP PROBLEM SOLVING
(continued)

3) Apply the principle of impulse and momentum in the x-direction from the time the log starts moving at $t_1 = 2.476$ s to $t_2 = 5$ s.

$$\xrightarrow{+} \quad mv_1 + \sum \int_{t_1}^{t_2} F\, dt = mv_2 \quad \text{where } v_1 = 0 \text{ at } t_1 = 2.476 \text{ s}$$

$$0 + \int_{2.476}^{5} T\, dt - \int_{2.476}^{5} \mu_k N\, dt = mv_2$$

$$\int_{2.476}^{4} 400t^2\, dt + \int_{4}^{5} 6400\, dt - \int_{2.476}^{5} (0.4)(4905)\, dt = (500)v_2$$

$$(400/3)t^3 \Big|_{2.476}^{4} + (6400)(5 - 4) – (0.4)(4905)(5 – 2.476) = (500)v_2$$

$$=> \quad v_2 = 15.9 \text{ m/s}$$

The kinetic coefficient of friction was used since the log is moving.