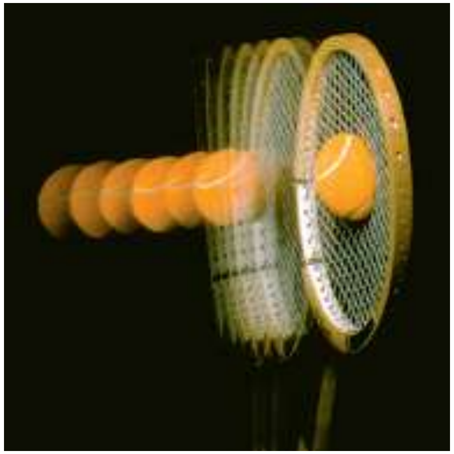
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES AND CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES

Today's Objectives:

Students will be able to:

1. Apply the principle of linear impulse and momentum to a system of particles.
2. Understand the conditions for conservation of momentum.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Linear Impulse And Momentum For A System of Particles
- Conservation of Linear Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The internal impulses acting on a system of particles always

   A) equal the external impulses.

   B) sum to zero.

   C) equal the impulse of weight.

   D) None of the above.

2. Weight is a(an)

   A) impulsive force.

   B) explosive force.

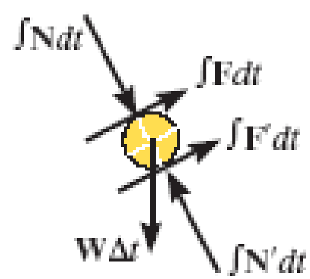
   C) non-impulsive force.

   D) internal force.

APPLICATIONS

As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of ***F*** dt and ***F'*** dt.

Does the release velocity of the ball depend on the mass of the ball?

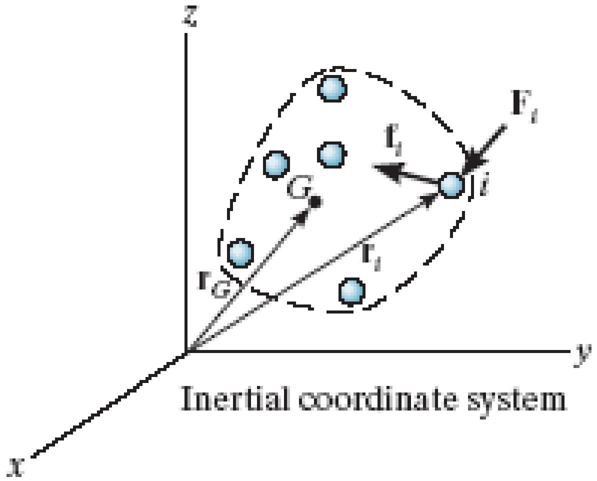


APPLICATIONS
(continued)

This large crane-mounted hammer is used to drive piles into the ground. Conservation of momentum can be used to find the velocity of the pile just after impact, assuming the hammer does not rebound off the pile.

If the hammer rebounds, does the pile velocity change from the case when the hammer doesn't rebound? Why?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES
(Section 15.2)



For the system of particles shown, the internal forces f_i between particles always occur in pairs with equal magnitude and opposite directions. Thus the internal impulses sum to zero.

The linear impulse and momentum equation for this system only includes the impulse of external forces.

$$\sum m_i(v_i)_1 + \sum \int_{t_1}^{t_2} F_i \, dt = \sum m_i(v_i)_2$$

MOTION OF THE CENTER OF MASS

For a system of particles, we can define a “fictitious” center of mass of an aggregate particle of mass m_{tot}, where m_{tot} is the sum ($\sum m_i$) of all the particles. This system of particles then has an aggregate velocity of $v_g = (\sum m_i v_i)/m_{tot}$.

The motion of this fictitious mass is based on motion of the center of mass for the system. The position vector $r_g = (\sum m_i r_i)/m_{tot}$ describes the motion of the center of mass.

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES

(Section 15.3)

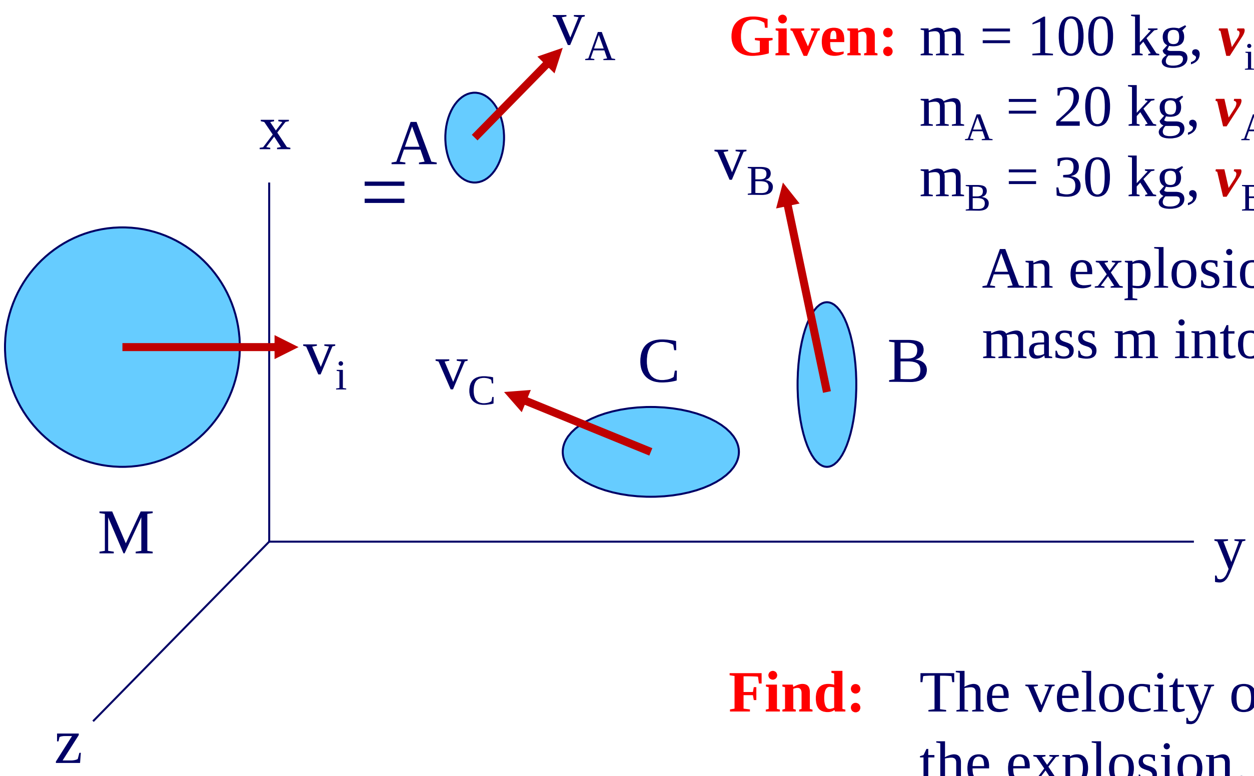
When the sum of external impulses acting on a system of objects is zero, the linear impulse-momentum equation simplifies to

$$\sum m_i(\boldsymbol{v}_i)_1 = \sum m_i(\boldsymbol{v}_i)_2$$

This important equation is referred to as the conservation of linear momentum. Conservation of linear momentum is often applied when particles collide or interact. When particles impact, only impulsive forces cause a change of linear momentum.

The sledgehammer applies an impulsive force to the stake. The weight of the stake can be considered negligible, or non-impulsive, as compared to the force of the sledgehammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground's reaction on the stake can also be considered negligible or non-impulsive.

EXAMPLE



Given: m = 100 kg, $\boldsymbol{v}_i = 20\boldsymbol{j}$ (m/s)
$m_A = 20$ kg, $\boldsymbol{v}_A = 50\boldsymbol{i} + 50\boldsymbol{j}$ (m/s)
$m_B = 30$ kg, $\boldsymbol{v}_B = -30\boldsymbol{i} - 50\boldsymbol{k}$ (m/s)

An explosion has broken the mass m into 3 smaller particles.

Find: The velocity of fragment C after the explosion.

Plan: Since the internal forces of the explosion cancel out, we can apply the conservation of linear momentum to the SYSTEM.

EXAMPLE
(continued)

Solution:

$$m\boldsymbol{v}_i = m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B + m_C\boldsymbol{v}_C$$

$$100(20\boldsymbol{j}) = 20(50\boldsymbol{i} + 50\boldsymbol{j}) + 30(-30\boldsymbol{i}-50\boldsymbol{k}) + 50(v_{cx}\,\boldsymbol{i} + v_{cy}\boldsymbol{j} + v_{cz}\,\boldsymbol{k})$$

Equating the components on the left and right side yields:

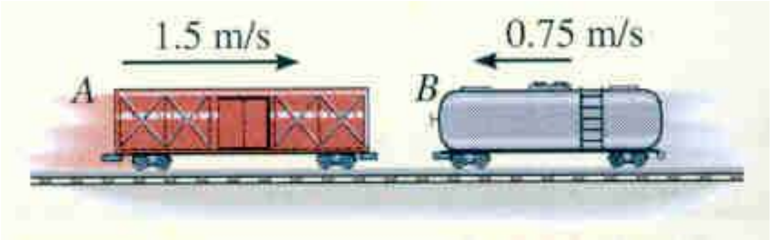
$0 = 1000 – 900 + 50(v_{cx})$ $\quad$ $v_{cx} = -2$ m/s

$2000 = 1000 + 50\ (v_{cy})$ $\quad$ $v_{cy} = 20$ m/s

$0 = -1500 + 50\ (v_{cz})$ $\quad$ $v_{cz} = 30$ m/s

So $\boldsymbol{v}_c = (-2\boldsymbol{i} + 20\boldsymbol{j} + 30\boldsymbol{k})$ m/s immediately after the explosion.

EXAMPLE II



Given: Two rail cars with masses of m_A = 15 Mg and m_B = 12 Mg and velocities as shown.

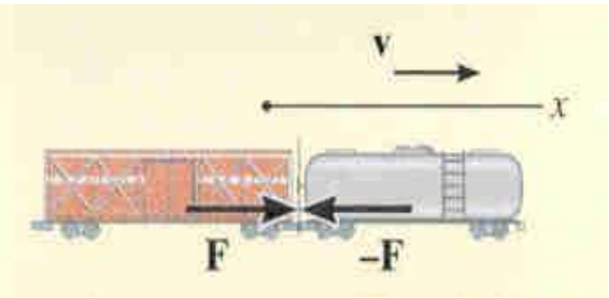
Find: The speed of the cars after they meet and connect. Also find the average impulsive force between the cars if the coupling takes place in 0.8 s.

Plan: Use conservation of linear momentum to find the velocity of the two cars after connection (all internal impulses cancel). Then use the principle of impulse and momentum to find the impulsive force by looking at only one car.

EXAMPLE II
(continued)

Solution:

Conservation of linear momentum (x-dir):

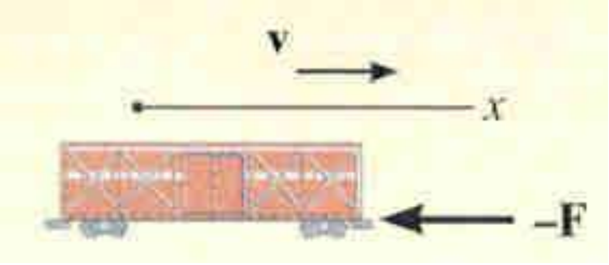


$$m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)\, v_2$$

$$15{,}000(1.5) + 12{,}000(-\,0.75) = (27{,}000)v_2$$

$$v_2 = 0.5 \text{ m/s}$$

Impulse and momentum on car A (x-dir):



$$m_A(v_A)_1 + \int F\,dt = m_A(v_2)$$

$$15{,}000(1.5) - \int F\,dt = 15{,}000(0.5)$$

$$\int F\,dt = 15{,}000 \text{ N·s}$$

The average force is

$$\int F\,dt = 15{,}000 \text{ N·s} = F_{avg}(0.8 \text{ sec}); \quad F_{avg} = 18{,}750 \text{ N}$$

CONCEPT QUIZ

1) Over the short time span of a tennis ball hitting the racket during a player's serve, the ball's weight can be considered

   A) nonimpulsive.

   B) impulsive.

   C) not subject to Newton's second law.

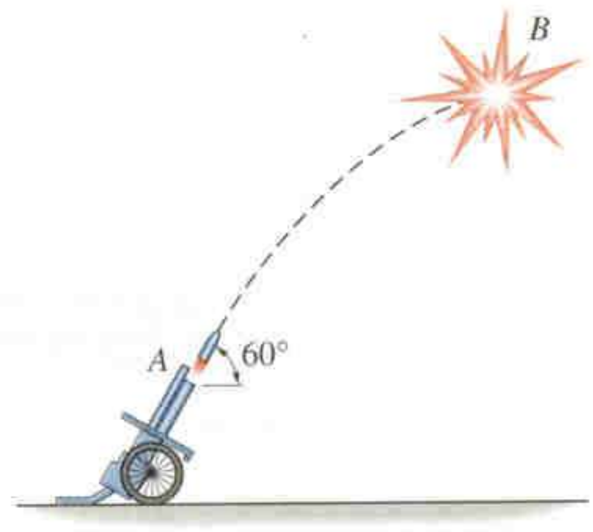
   D) Both A and C.

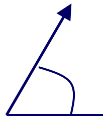
2) A drill rod is used with a air hammer for making holes in hard rock so explosives can be placed in them. How many impulsive forces act on the drill rod during the drilling?

   A) None B) One

   C) Two D) Three

GROUP PROBLEM SOLVING



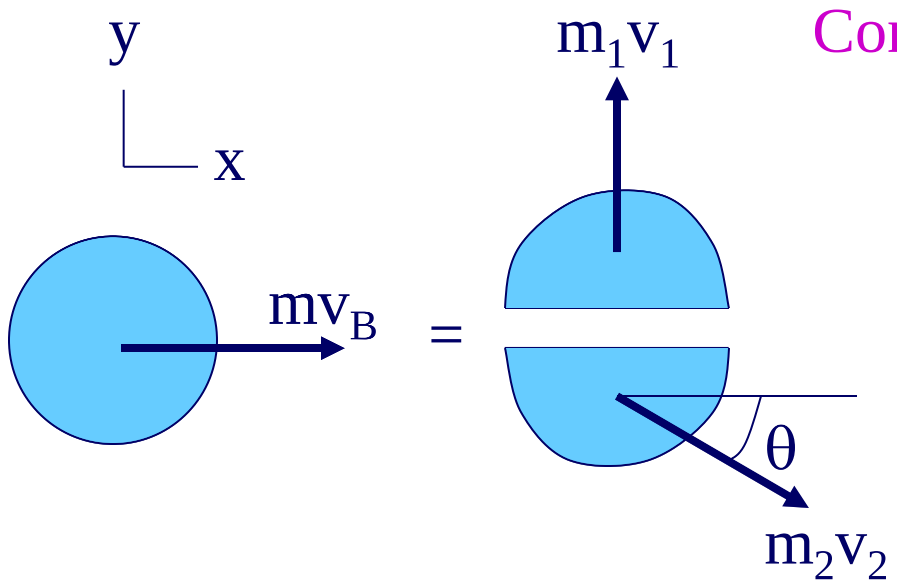
Given: A 10-lb projectile is fired from point A. It's velocity is 80 ft/s @  60°
The projectile explodes at its highest point, B, into two 5-lb fragments. One fragment moves vertically upward at v_y = 12 ft/s.

Find: Determine the velocity of the other fragment immediately after the explosion.

Plan: Since we know $(v_B)_y$ = 0 just before the explosion, we can determine the velocity of the projectile fragments immediately after the explosion.

GROUP PROBLEM SOLVING
(continued)

Solution:



Conservation of linear momentum:

Since the impulse of the explosion is an internal impulse, the system's linear momentum is conserved. So

$$m\boldsymbol{v}_B = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

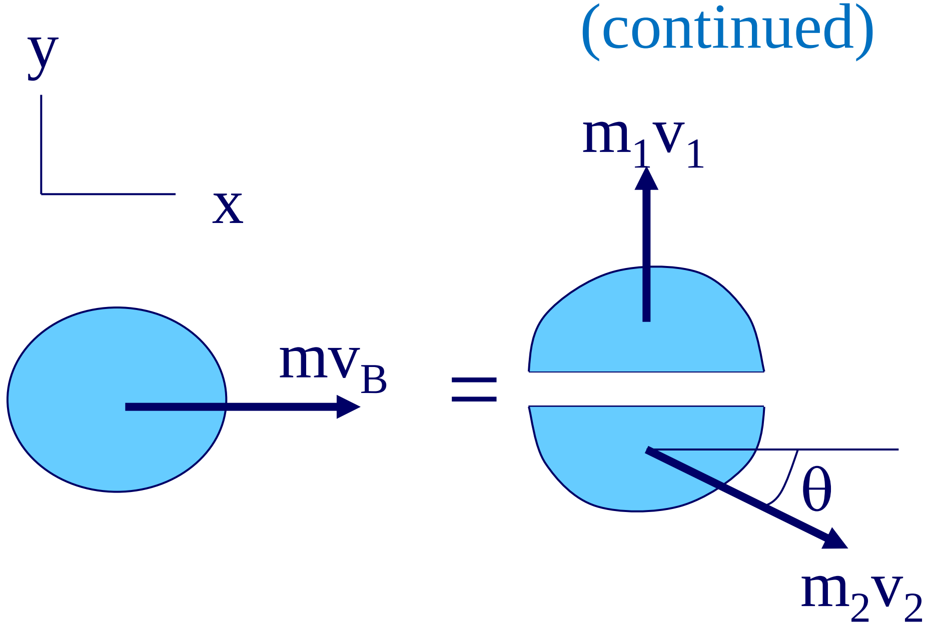
We know $(v_B)_y = 0$. Use projectile motion equations to calculate $(v_B)_x$:

$$(v_B)_x = (v_A)_x = v_A \cos 60° = 80 \cos 60° = 40 \text{ ft/s}$$

Therefore, substituting into the linear momentum equation

$$(10/g)(40)\,\boldsymbol{i} = (5/g)(12)\,\boldsymbol{j} + (5/g)(v_2)(\cos\theta\,\boldsymbol{i} - \sin\theta\,\boldsymbol{j})$$

GROUP PROBLEM SOLVING
(continued)



$$(10/g)(40)\,\boldsymbol{i} = (5/g)(12)\,\boldsymbol{j} + (5/g)(v_2)(\cos\theta\,\boldsymbol{i} - \sin\theta\,\boldsymbol{j})$$

Eliminating g, dividing by 5 and creating ***i*** & ***j*** component equations yields: $80 = v_2 \cos\theta$

$$0 = 12 - v_2 \sin\theta$$

Solving for v_2 and θ yields $\theta = 8.53°$ and $v_2 = 80.9$ ft/s

ATTENTION QUIZ

1. The 20-g bullet is fired horizontally at 1200 m/s into the 300-g block resting on a smooth surface. If the bullet becomes embedded in the block, what is the velocity of the block immediately after impact.

A) 1125 m/s B) 80 m/s

C) 1200 m/s D) 75 m/s

2. The 200-g baseball has a horizontal velocity of 30 m/s when it is struck by the bat, B, weighing 900-g, moving at 47 m/s. During the impact with the bat, how many impulses of importance are used to find the final velocity of the ball?

A) Zero B) One

C) Two D) Three