CONSERVATIVE FORCES AND POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Today's Objectives:

Students will be able to:

1. Understand the concept of conservative forces and determine the potential energy of such forces.
2. Apply the principle of conservation of energy.

In-Class Activities:

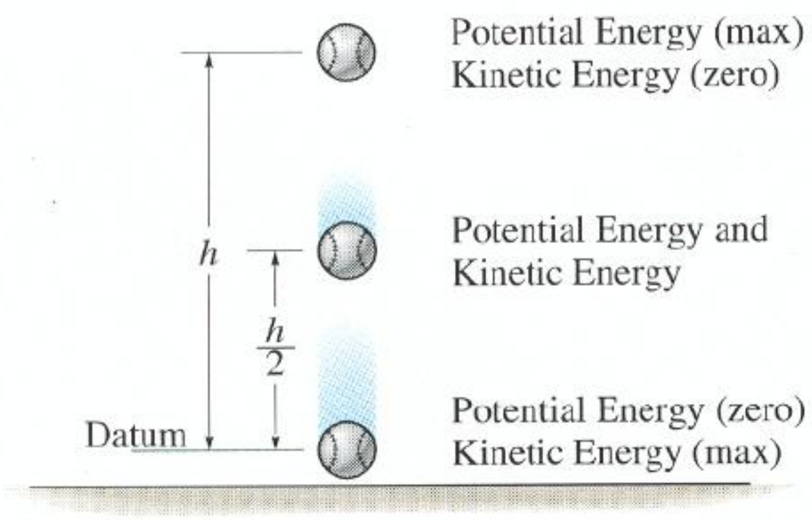
- Check Homework
- Reading Quiz
- Applications
- Conservative Force
- Potential Energy
- Conservation of Energy
- Concept Quiz
- Group Problem Solving
- Attention Quiz

APPLICATIONS

The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

If the sacks weigh 100 *lb* and the equivalent spring constant is $k = 500$ *lb/ft*, what is the energy stored in the springs?

APPLICATIONS
(continued)



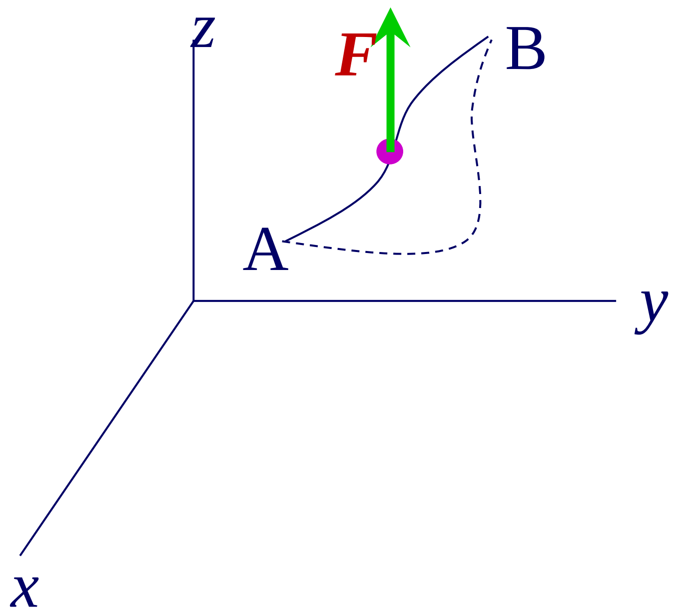
When a ball of weight *W* is dropped (from rest) from a height *h* above the ground, the potential energy stored in the ball is converted to kinetic energy as the ball drops.

What is the velocity of the ball when it hits the ground? Does the weight of the ball affect the final velocity?

CONSERVATIVE FORCE
(Section 14.5)

A force ***F*** is said to be conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B. In other words, the work done by the force ***F*** in a closed path (*i.e.*, from A to B and then back to A) equals zero.

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$



This means the work is conserved.

A conservative force depends only on the position of the particle, and is independent of its velocity or acceleration.

CONSERVATIVE FORCE
(continued)

A more rigorous definition of a conservative force makes use of a potential function (V) and partial differential calculus, as explained in the texts. However, even without the use of the these mathematical relationships, much can be understood and accomplished.

The "conservative" potential energy of a particle/system is typically written using the potential function V. There are two major components to V commonly encountered in mechanical systems, the potential energy from gravity and the potential energy from springs or other elastic elements.

$$V_{\text{total}} = V_{\text{gravity}} + V_{\text{springs}}$$

POTENTIAL ENERGY

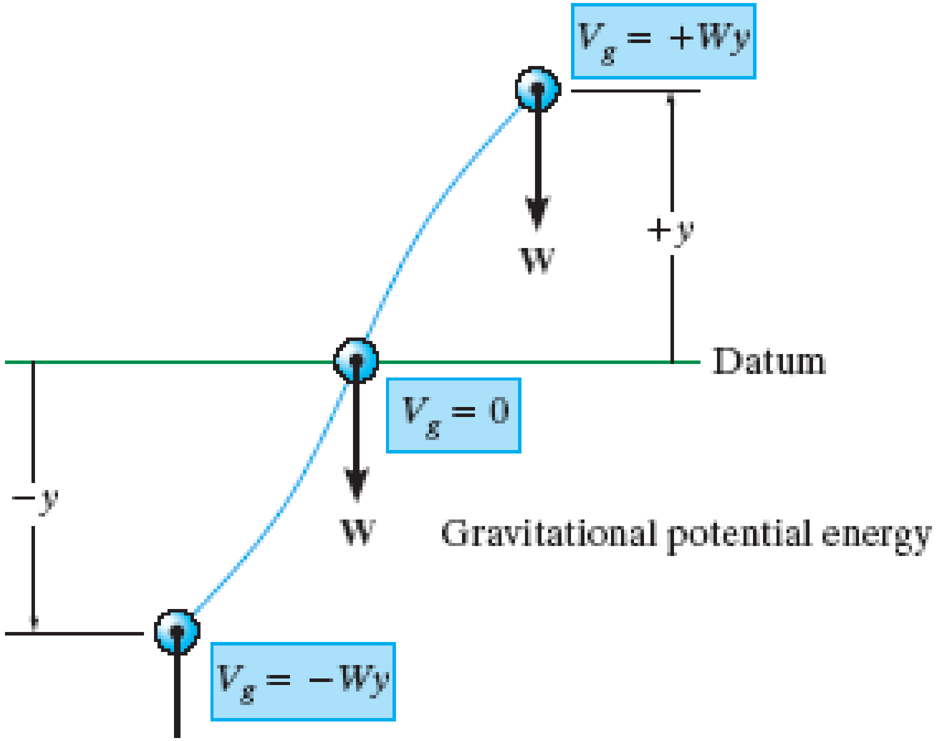
Potential energy is a measure of the amount of work a conservative force will do when it changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (the sum of $V_{gravity}$ and $V_{springs}$).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

POTENTIAL ENERGY DUE TO GRAVITY

The potential function (formula) for a gravitational force, e.g., weight (W = mg), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

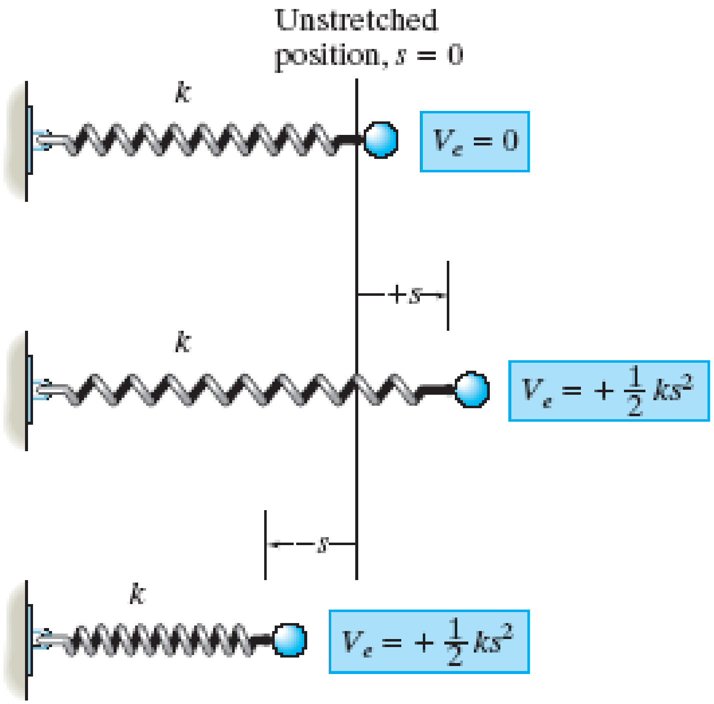


$$V_g = \pm W y$$

V_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum.

ELASTIC POTENTIAL ENERGY

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.



Elastic potential energy

V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2}ks^2$$

Notice that the potential function V_e always yields positive energy.

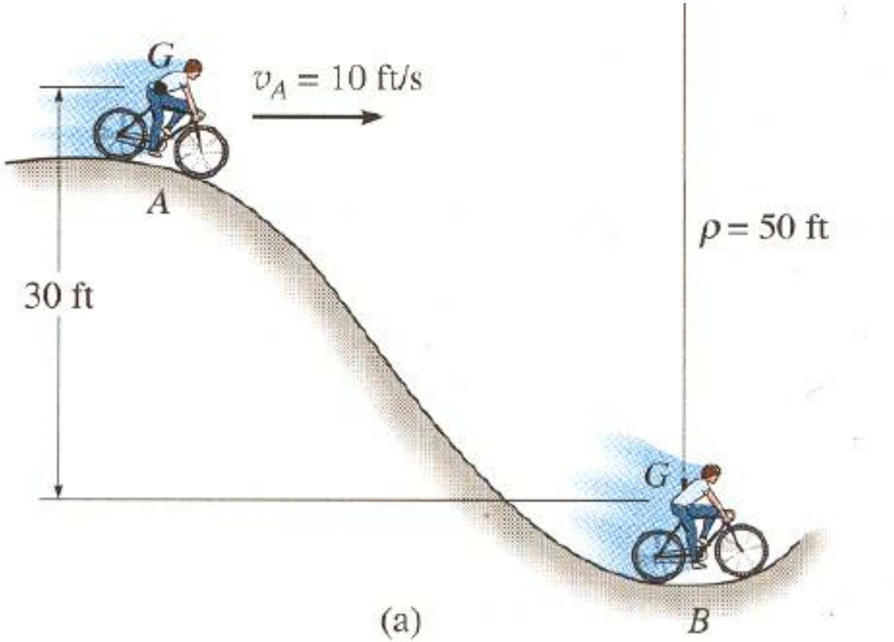
CONSERVATION OF ENERGY
(Section 14.6)

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 \quad = \text{Constant}$$

T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_2 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = ½\, mv^2$.

EXAMPLE



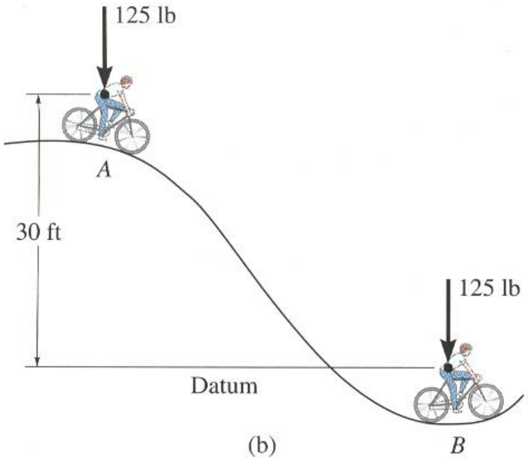
Given: The girl and bicycle weigh 125 lbs. She moves from point A to B.

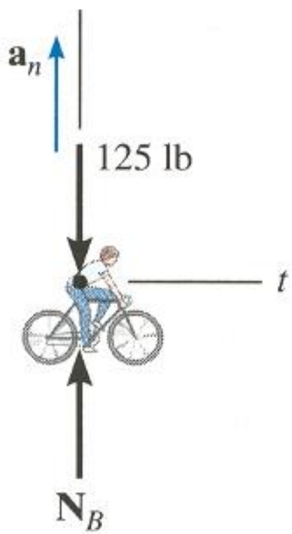
Find: The velocity and the normal force at B if the velocity at A is 10 ft/s and she stops pedaling at A.

Plan: Note that only kinetic energy and potential energy due to gravity (V_g) are involved. Determine the velocity at B using the conservation of energy equation and then apply equilibrium equations to find the normal force.

EXAMPLE
(continued)

Solution:





Placing the datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}\left(\frac{125}{32.2}\right)(10)^2 + 125(30) = \frac{1}{2}\left(\frac{125}{32.2}\right)v_B^2$$

$$V_B = 45.1 \ {ft}/{s}$$

Equation of motion applied at B:

$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

$$N_B - 125 = \frac{125}{32.2}\,\frac{(45.1)^2}{50}$$

$$N_B = 283\,\text{lb}$$

CONCEPT QUIZ

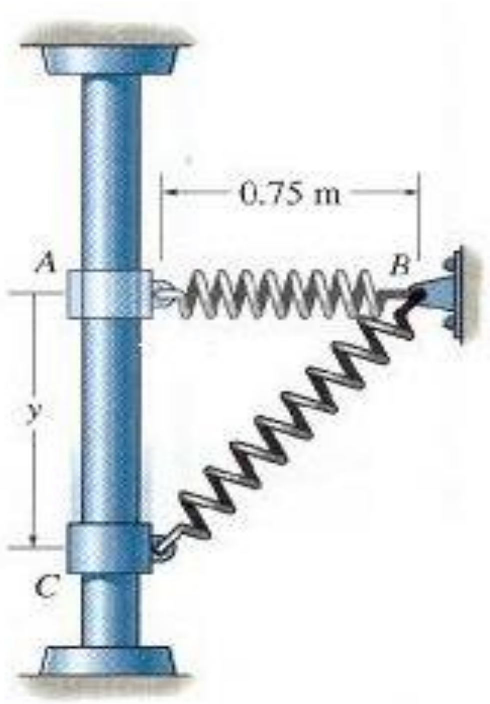
1. If the work done by a conservative force on a particle as it moves between two positions is –10 ft-lb, the change in its potential energy is

   A) 0 ft-lb.
   B) -10 ft-lb.
   C) +10 ft-lb.
   D) None of the above.

2. Recall that the work of a spring is $U_{1-2} = -\frac{1}{2} k(s_2^2 - s_1^2)$ and can be either positive or negative. The potential energy of a spring is $V = \frac{1}{2} ks^2$. Its value is

   A) always negative.
   B) either positive or negative.
   C) always positive.
   D) an imaginary number!

GROUP PROBLEM SOLVING



Given: The mass of the collar is 2 kg and the spring constant is 60 N/m. The collar has no velocity at A and the spring is un-deformed at A.

Find: The maximum distance y the collar drops before it stops at Point C.

Plan: Apply the conservation of energy equation between A and C. Set the gravitational potential energy datum at point A or point C (in this example, choose point A).

GROUP PROBLEM SOLVING
(continued)

Solution:

Notice that the potential energy at *C* has two parts ($T_c = 0$).

$$V_c = (V_c)_e + (V_c)_g$$

Placing the datum for gravitational potential at *A* yields a conservation of energy equation with the left side all zeros. Since T_c equals zero at points A and C, the equation becomes

$$0 + 0 = 0 + [\frac{60}{2}(\sqrt{(.75)^2 + y^2} - .75)^2 - 2(9.81)y]$$

Note that $(V_c)_g$ is negative since point C is below the datum.

Since the equation is nonlinear, a numerical solver can be used to find the solution or root of the equation. This solving routine can be done with a calculator or a program like Excel. The solution yields y = 1.61 m.

GROUP PROBLEM SOLVING
(continued)

Also notice that since the velocities at *A* and *C* are zero, the velocity must reach a maximum somewhere between *A* and *C*.

Since energy is conserved, the point of maximum kinetic energy (maximum velocity) corresponds to the point of minimum potential energy.

By expressing the potential energy at any given position as a function of *y* and then differentiating, we can determine the position at which the velocity is maximum (since $dV/dy = 0$ at this position). The derivative yields another nonlinear equation which could be solved using a numerical solver.

ATTENTION QUIZ

1. The principle of conservation of energy is usually _______ to apply than the principle of work & energy.

   A) harder B) easier

   C) the same amount of work D) Don't pick this one.

2. If the pendulum is released from the horizontal position, the velocity of its bob in the vertical position is

   A) 3.8 m/s . B) 6.9 m/s.

   C) 14.7 m/s. D) 21 m/s.