# INTRODUCTION & RECTILINEAR KINEMATICS: CONTINUOUS MOTION

**Today's Objectives:**

Students will be able to:

1. Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a straight path.

Rectilinear means position given in Cartesian (*x*, *y*, and *z*) coordinates.

We will stat with motion in a straight line.

# An Overview of Mechanics

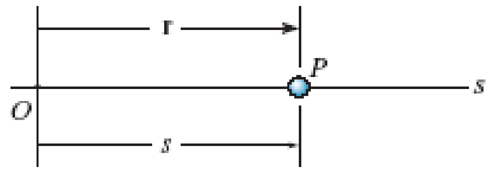
Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

***Dynamics:***

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** - concerned with the forces causing the motion

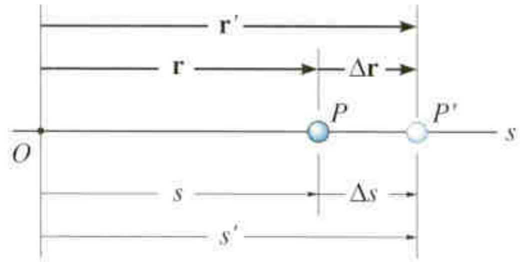
# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)



Position

A particle travels along a straight-line path defined by the coordinate axis s.

The position of the particle at any instant, relative to the origin, O, is defined by the position vector ***r***, or the scalar s. Scalar s can be positive or negative. Typical units for ***r*** and s are meters (m) or feet (ft).



Displacement

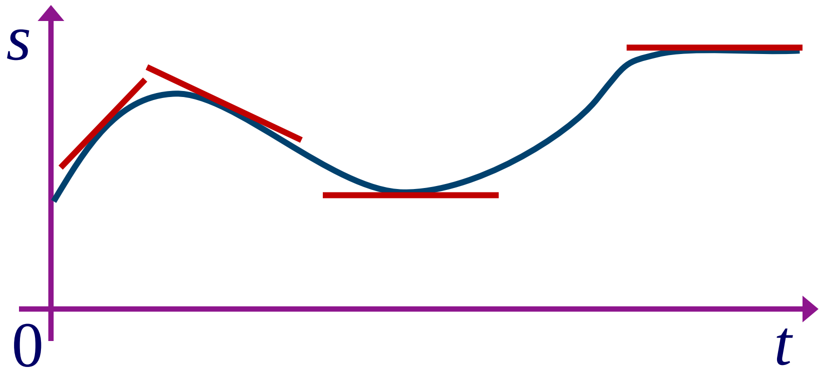
The displacement of the particle is defined as its change in position.

Vector form: $\boldsymbol{\Delta r = r' - r}$ Scalar form: $\Delta s = s' - s$

The total distance traveled by the particle, $s_T$, is a positive scalar that represents the total length of the path over which the particle travels.

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

The easiest way to study the motion of a particle is to graph position versus time.



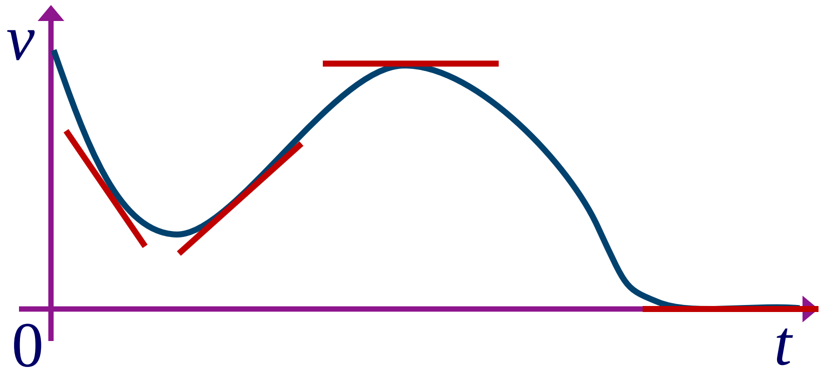

We can define velocity *v* as the slope of a line tangent to *s*-*t* curve.

$$v = \frac{ds}{dt}$$

Positive slope → v > 0 → particle moving in positive direction.
Negative slope → v < 0 → particle moving in negative direction.
Zero slope → v = 0 → particle turning around OR stopped.

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

The easiest way to study the velocity of a particle is to graph velocity versus time.



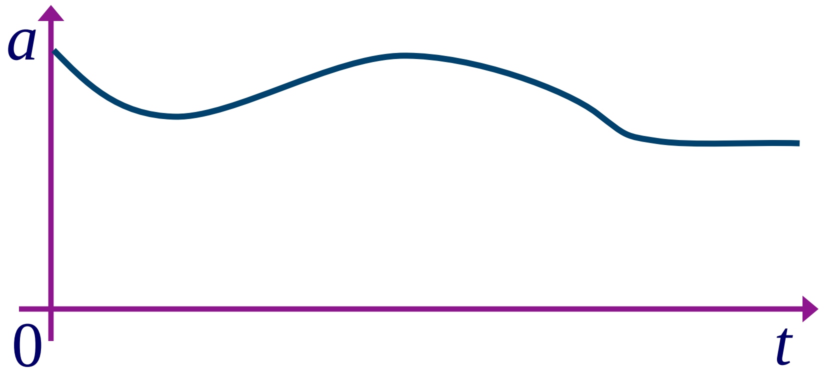

We now define acceleration *a* as the slope of tangent to *v*-*t* curve.

$$a = \frac{dv}{dt}$$

If *a* and *v* are both the same sign, object is speeding up.
If *a* and *v* are opposite sign, object is slowing.

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We can study the acceleration of a particle by a graph as well.




But we usually don't need to look at further derivatives.

Given the functional dependence of s(t) or v(t) we can use the methods of Calculus I to find derivatives.

# Example

Given $s(t) = 7t^3 + 5t + 2$, find $v(t)$ and $a(t)$.

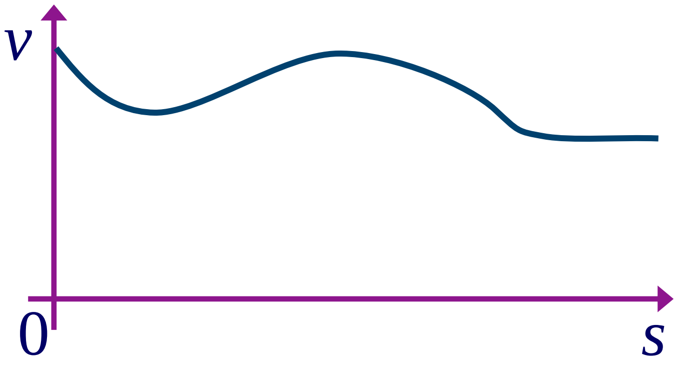
$$v(t) = \frac{ds}{dt} = 21t^2 + 5.$$

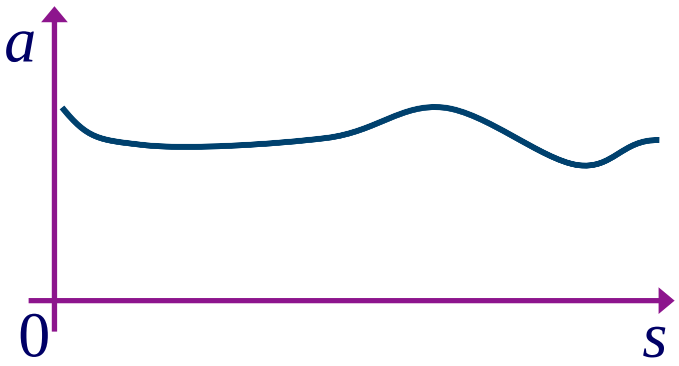
And

$$a(t) = \frac{dv}{dt} = 42t.$$

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We are so used to studying position, velocity, and acceleration versus time curves that we forget that we can also study how velocity and acceleration depend on *position*!

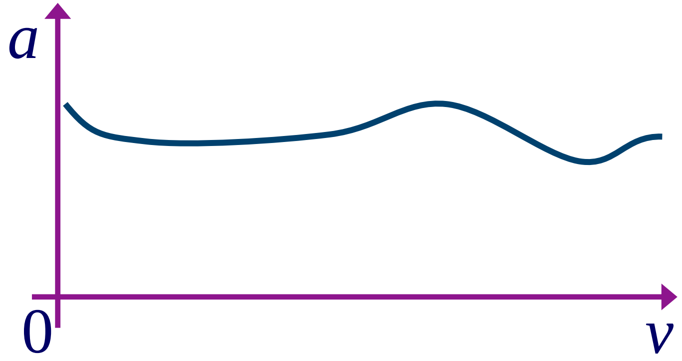



**NOTE!** $a(s) \neq \frac{dv}{ds}$

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION
## (Section 12.2)

We can also study how acceleration depend on *velocity*!



# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We have six motion functions:

$$s(t)$$

$$v(t) \qquad v(s)$$

$$a(t) \qquad a(s) \qquad a(v)$$

We have two relational definitions:

$$v = \frac{ds}{dt} \qquad \text{and} \qquad a = \frac{dv}{dt}$$

**Goal**. Given one motion function, find the others!

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We know the operational meaning of our definitions, $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$, that is how to differentiate $s(t)$ and $v(t)$.

We often fail to recognize that our definitions, $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$, can be treated like ordinary equations involving fractions that can be manipulated as we choose.

So for instance $v = \frac{ds}{dt}$ can be rearranged as $dt = \frac{ds}{v}$.

And this result $dt = \frac{ds}{v}$ can be substituted into $a = \frac{dv}{dt}$ to yield a new relationship $a = \frac{dv}{ds/v} = v\frac{dv}{ds}$ which tells us how to get $a(s)$ and $v(s)$.

# Example

Given $v(s) = 5s + 2$, find $a(s)$.

$$a(s) = v\frac{dv}{ds} = (5s + 2)\,5 = 25s + 10.$$

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We have six motion functions:

$$s(t)$$

$$v(t) \qquad v(s)$$

$$a(t) \qquad a(s) \qquad a(v)$$

We have **three** relational definitions:

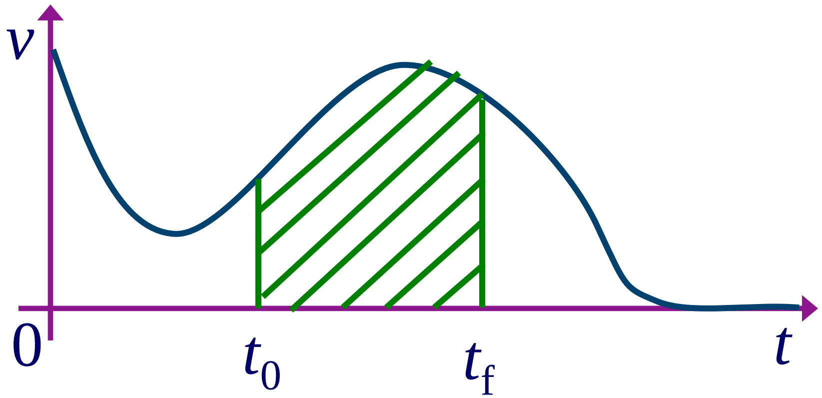
$$v = \frac{ds}{dt}\ , \quad a = \frac{dv}{dt}\, , \ \text{and } a = v\frac{dv}{ds}$$

We use differential calculus to move down the list of functions.

What if we want to move up the list?

# RECTILINEAR KINEMATICS: CONTINUOUS MOTION (Section 12.2)

We integrate!




Displacement $\Delta s = \int_{t_0}^{t_f} v\, dt$ Area under the curve

Since $\Delta s = s_f - s_0$, if we know the initial condition $s_0$ at $t_0$, we have $s_f$.

Usually $v_f$, $s_f$, and $t_f$ are just written as $v$, $s$, and $t$ if there is no chance of confusion.

# Method of Separation of Variables

Suppose we are asked to find $v(s = 5\text{ m})$ given $a(s) = 5s + 2$ and the initial condition that $v_0 = 4$ m/s at $s_0 = 0$.

Step 1. Find the differential relationship that has $v$, $a$, and $s$

Here it is $a = v\frac{dv}{ds}$

Step 2. Rearrange the differential relationship so variables of the same type are on the same side of the equals

$$a(s)\,ds = vdv$$

Step 3. Since both sides are equal, the integrals of the sides must also equal

$$\int_{s_0=0}^{s} a(s)\,ds = \int_{v_0=4}^{v} vdv$$

*Note:* even though we know $s = 5$ m we first get $v$ as a general function of $s$..

# Method of Separation of Variables

$$\int_0^s (5s + 2)\, ds = \int_4^v v\,dv$$

$$\frac{5}{2}s^2 + 2s \,\Big|_0^s = \frac{1}{2}v^2 \,\Big|_4^v$$

$$\frac{5}{2}s^2 + 2s = \frac{1}{2}v^2 - 8$$

$$v^2 = 5s^2 + 4s + 8$$

$$v = \pm\sqrt{5s^2 + 4s + 8}$$

When $s = 5$, $v = \pm\sqrt{153}$. Note two possible solutions!

Since $a > 0$ for all $s$, and $v_0 > 0$, the solution is $v = +\sqrt{153}$

# SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

- Differentiate position to get velocity and acceleration.

$$v = ds/dt \; ; \qquad a = dv/dt \quad \text{or} \quad a = v\, dv/ds$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\, dt \quad \text{or} \quad \int_{v_o}^{v} v\, dv = \int_{s_o}^{s} a\, ds$$

Position:

$$\int_{s_o}^{s} ds = \int_{o}^{t} v\, dt$$

- Note that $s_o$ and $v_o$ represent the initial position and velocity of the particle at $t = 0$.

# CONSTANT ACCELERATION

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$) to obtain very useful equations. A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2$ downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c \, dt \quad \text{yields} \quad v = v_o + a_c t$$

$$\int_{s_o}^{s} ds = \int_{o}^{t} v \, dt \quad \text{yields} \quad s = s_o + v_o t + (1/2) a_c t^2$$

$$\int_{v_o}^{v} v \, dv = \int_{s_o}^{s} a_c \, ds \quad \text{yields} \quad v^2 = (v_o)^2 + 2a_c(s - s_o)$$

# EXAMPLE

**Given:** A motorcyclist travels along a straight road at a speed of 27 m/s. When the brakes are applied, the motorcycle decelerates at a rate of -6t $m/s^2$.

**Find:** The distance the motorcycle travels before it stops.

**Plan:** Establish the positive coordinate s in the direction the motorcycle is traveling. Since the acceleration is given as a function of time, integrate it once to calculate the velocity and again to calculate the position.

# EXAMPLE

**Solution:** (continued)

1) Integrate acceleration to determine the velocity.

$$a = dv / dt \quad \Rightarrow \quad dv = a\,dt \quad \Rightarrow \quad \int_{v_o}^{v} dv = \int_{o}^{t} (-6t)\,dt$$

$$\Rightarrow \quad v - v_o = -3t^2 \quad \Rightarrow \quad v = -3t^2 + v_o$$

2) We can now determine the amount of time required for the motorcycle to stop ($v = 0$). Use $v_o = 27$ m/s.

$$0 = -3t^2 + 27 \quad \Rightarrow \quad t = 3 \text{ s}$$

3) Now calculate the distance traveled in 3s by integrating the velocity using $s_o = 0$:

$$v = ds / dt \quad \Rightarrow \quad ds = v\,dt \quad \Rightarrow \quad \int_{s_o}^{s} ds = \int_{o}^{t} (-3t^2 + v_o)\,dt$$

$$\Rightarrow \quad s - s_o = -t^3 + v_o t$$

$$\Rightarrow \quad s - 0 = (3)^3 + (27)(3) \quad \Rightarrow \quad s = 54 \text{ m}$$

## EXAMPLE

**Given:** The acceleration of a body is $a = 5v$. At $t = 0$, $s = 0$, and $v_0 = 2$ m/s. (a) Find v(t). (b) Find v(s).

(a) Need the equation with $a$, $v$, and $t$.

$$a = \frac{dv}{dt}$$

Rearrange. Terms with $t$ on one side and $v$ on the other.

$$dt = \frac{dv}{a}$$

Create integrals

$$\int_0^t dt = \int_2^v \frac{dv}{5v}$$

## EXAMPLE (continued)

Result is

$$t\Big|_0^t = \frac{1}{5}\ln(v)\Big|_2^v$$

$$t = \frac{1}{5}\ln\left(\frac{v}{2}\right)$$

$$v = 2e^{5t}$$

(b) Need the equation with *a*, *v*, and *s*.

$$a = v\frac{dv}{ds}$$

$$ds = v\frac{dv}{a}$$

## EXAMPLE (continued)

Result is

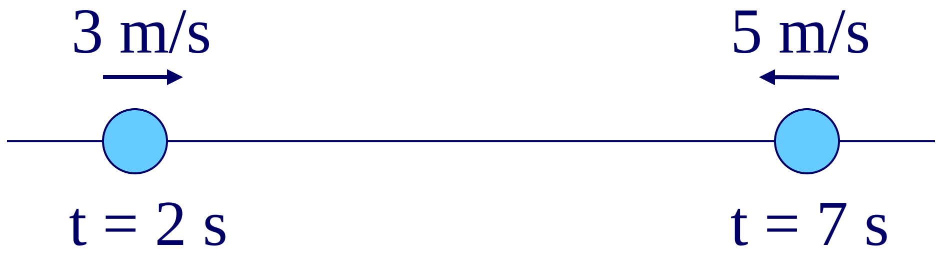
$$\int_0^s ds = \int_2^v v\frac{dv}{5v}$$

$$s\Big|_0^s = \frac{1}{5}v\Big|_2^v$$

$$s = \frac{1}{5}(v-2)$$

$$v = 5s + 2$$

# CONCEPT QUIZ




1. A particle moves along a horizontal path with its velocity varying with time as shown. The average acceleration of the particle is __________.

   A) 0.4 $m/s^2$ →    B) 0.4 $m/s^2$ ←

   C) 1.6 $m/s^2$ →    D) 1.6 $m/s^2$ ←

2. A particle has an initial velocity of 30 ft/s to the left. If it then passes through the same location 5 seconds later with a velocity of 50 ft/s to the right, the average velocity of the particle during the 5 s time interval is _______.

   A) 10 ft/s →    B) 40 ft/s →

   C) 16 m/s →    D) 0 ft/s

# ATTENTION QUIZ

1. A particle has an initial velocity of 3 ft/s to the left at $s_0 = 0$ ft. Determine its position when t = 3 s if the acceleration is 2 ft/$s^2$ to the right.

   A) 0.0 ft
   B) 6.0 ft ←
   C) 18.0 ft →
   D) 9.0 ft →

2. A particle is moving with an initial velocity of v = 12 ft/s and constant acceleration of 3.78 ft/$s^2$ in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 ft/s.

   A) 50 ft
   B) 100 ft
   C) 150 ft
   D) 200 ft