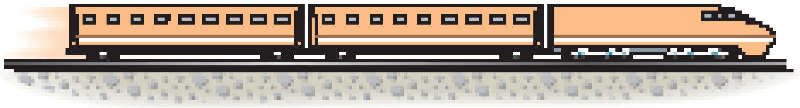
RECTILINEAR KINEMATICS: ERRATIC MOTION

Today's Objectives:

Students will be able to:

1. Determine position, velocity, and acceleration of a particle using graphs.



In-Class Activities:

- Reading Quiz
- Applications
- s-t, v-t, a-t, v-s, and a-s diagrams
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The slope of a v-t graph at any instant represents instantaneous

    A) velocity. B) acceleration.

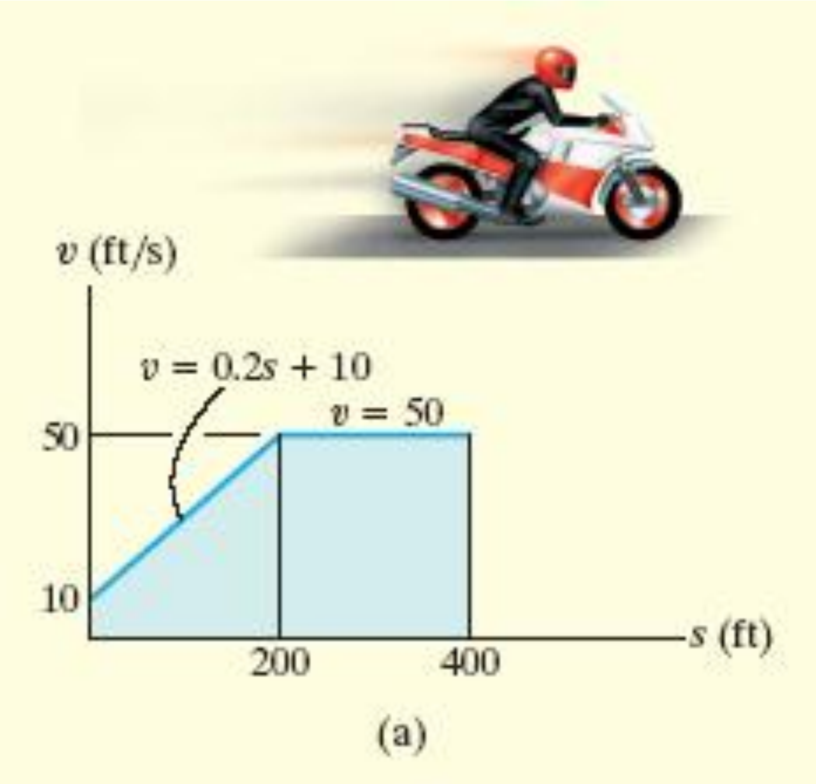
    C) position. D) jerk.

2. Displacement of a particle in a given time interval equals the area under the ___ graph during that time.

    A) a-t B) a-s

    C) v-t C) s-t

APPLICATION



In many experiments, a velocity versus position (v-s) profile is obtained.

If we have a v-s graph for the motorcycle, can we determine its acceleration at position s = 150 feet?

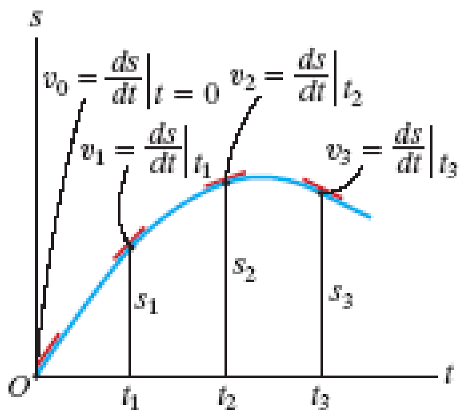
How?

ERRATIC MOTION
(Section 12.3)

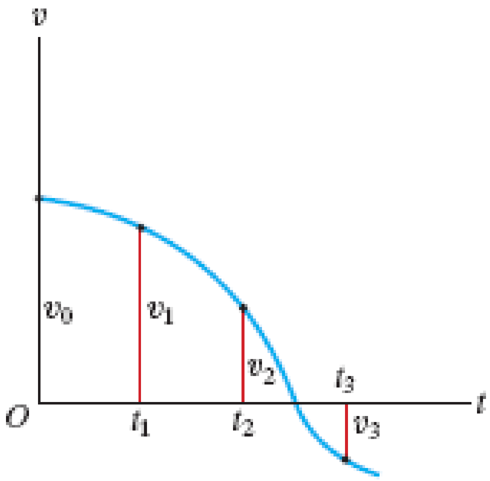
Graphing provides a good way to handle complex motions that would be difficult to describe with formulas. Graphs also provide a visual description of motion and reinforce the calculus concepts of differentiation and integration as used in dynamics.

The approach builds on the facts that slope and differentiation are linked and that integration can be thought of as finding the area under a curve.

s-t GRAPH

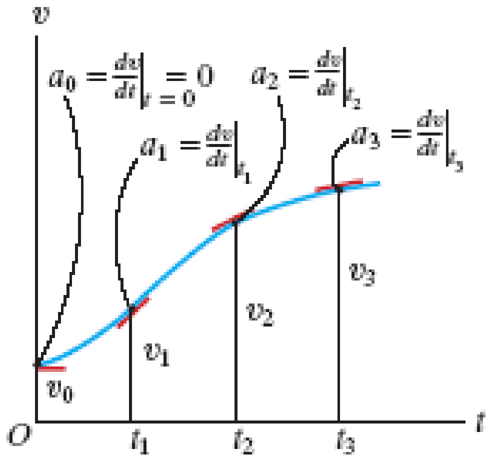


Plots of position vs. time can be used to find velocity vs. time curves. Finding the slope of the line tangent to the motion curve at any point is the velocity at that point (or v = ds/dt).



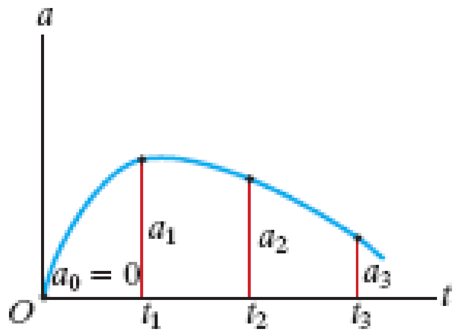
Therefore, the v-t graph can be constructed by finding the slope at various points along the s-t graph.

v-t GRAPH



Plots of velocity vs. time can be used to find acceleration vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point (or a = dv/dt).

Therefore, the a-t graph can be constructed by finding the slope at various points along the v-t graph.



Also, the distance moved (displacement) of the particle is the area under the v-t graph during time Δt.

a-t GRAPH

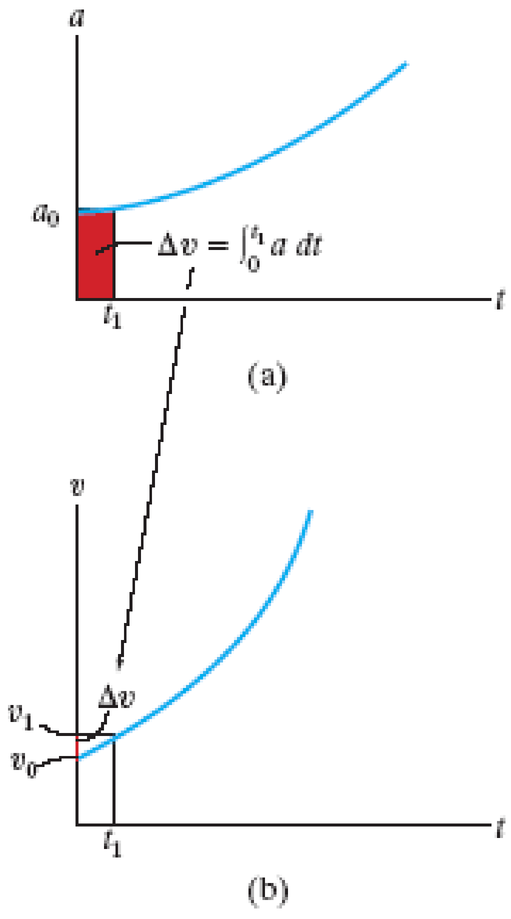


Fig. 12–10

Given the a-t curve, the change in velocity (Δv) during a time period is the area under the a-t curve.

So we can construct a v-t graph from an a-t graph if we know the initial velocity of the particle.

a-s GRAPH

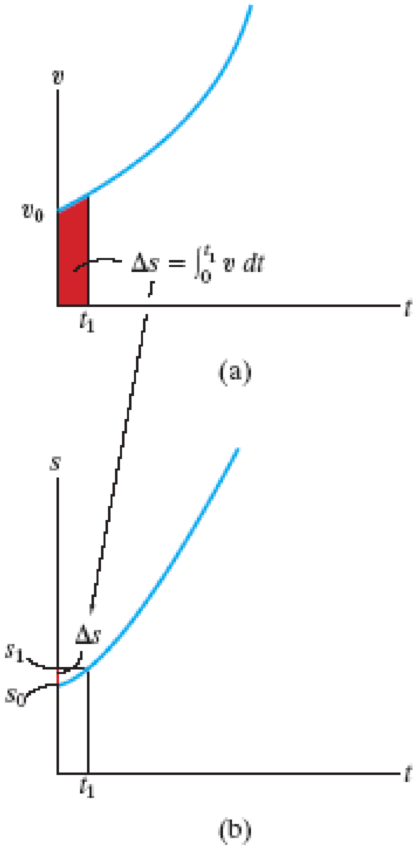


Fig. 12–11

A more complex case is presented by the a-s graph. The area under the acceleration versus position curve represents the change in velocity (recall $\int$ a ds = $\int$ v dv).

$$\tfrac{1}{2}\,(v_1^2 - v_o^2) = \int_{s_1}^{s_2} a\, ds \;=\; \text{area under the a-s graph}$$

This equation can be solved for v_1, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance.

v-s GRAPH

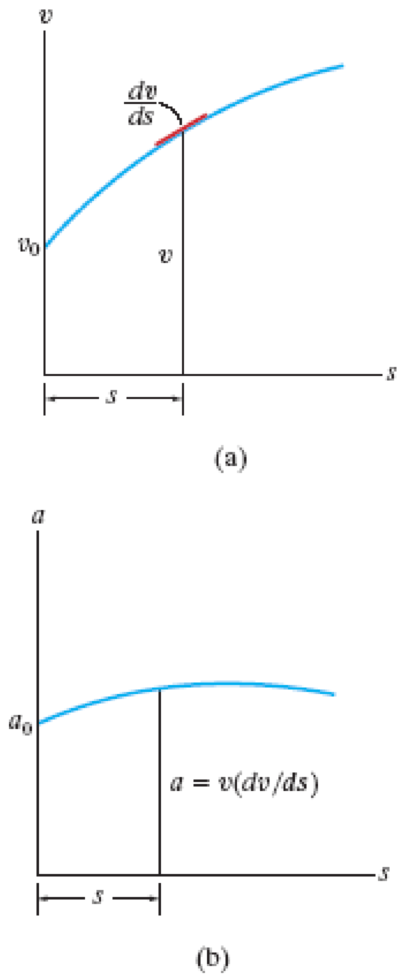


Fig. 12–14

Another complex case is presented by the v-s graph. By reading the velocity v at a point on the curve and multiplying it by the slope of the curve (dv/ds) at this same point, we can obtain the acceleration at that point.

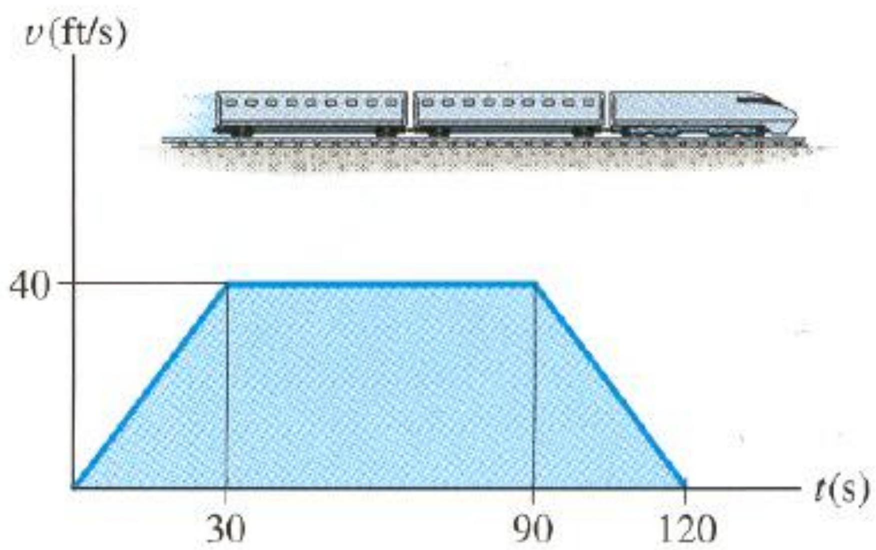
$$a = v\,(dv/ds)$$

Thus, we can obtain a plot of a vs. s from the v-s curve.

EXAMPLE

Given: v-t graph for a train moving between two stations

Find: a-t graph and s-t graph over this time interval



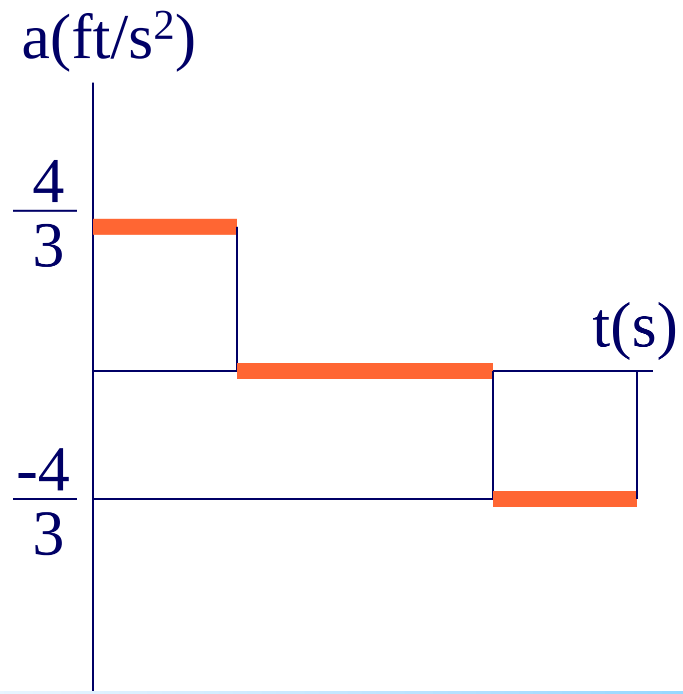
Think about your plan of attack for the problem!

EXAMPLE (continued)

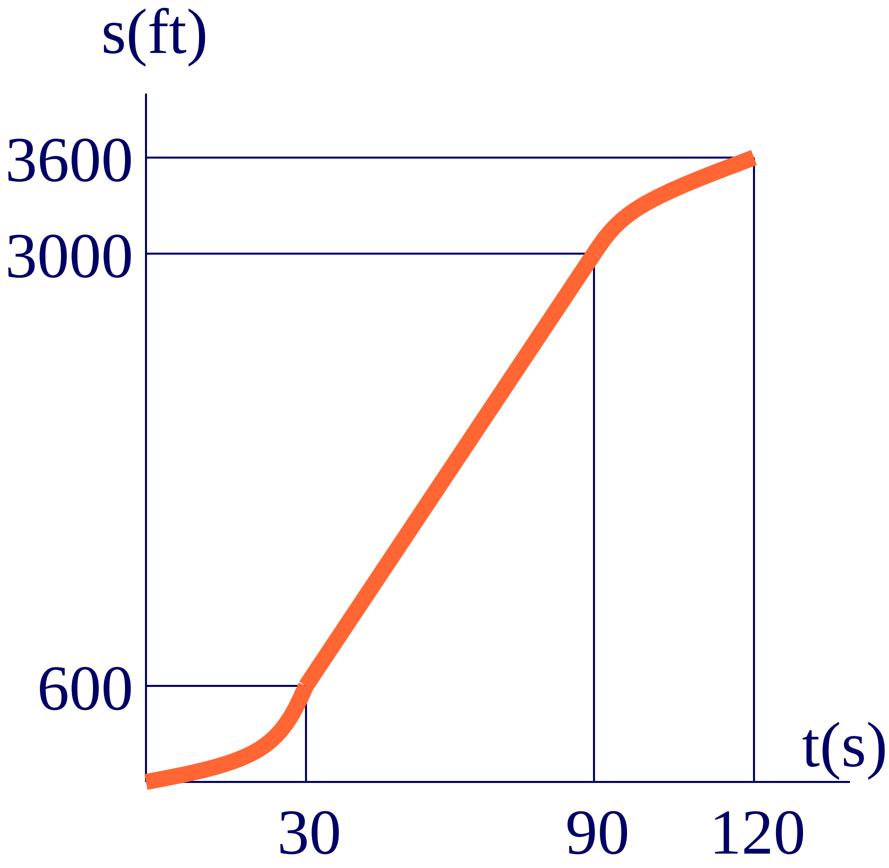
Solution: For the first 30 seconds the slope is constant and is equal to:

$$a_{0\text{-}30} = dv/dt = 40/30 = 4/3 \text{ ft/s}^2$$

Similarly, $a_{30\text{-}90} = 0$ and $a_{90\text{-}120} = -4/3 \text{ ft/s}^2$



EXAMPLE
(continued)



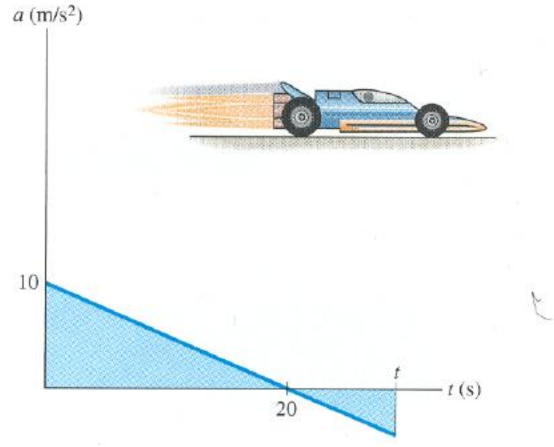
The area under the v-t graph represents displacement.

$\Delta s_{0\text{-}30} = \frac{1}{2}(40)(30) = 600$ ft

$\Delta s_{30\text{-}90} = (60)(40) = 2400$ ft

$\Delta s_{90\text{-}120} = \frac{1}{2}(40)(30) = 600$ ft

CONCEPT QUIZ



1. If a particle starts from rest and accelerates according to the graph shown, the particle's velocity at t = 20 s is

A) 200 m/s

B) 100 m/s

C) 0

D) 20 m/s

2. The particle in Problem 1 stops moving at t = _______.

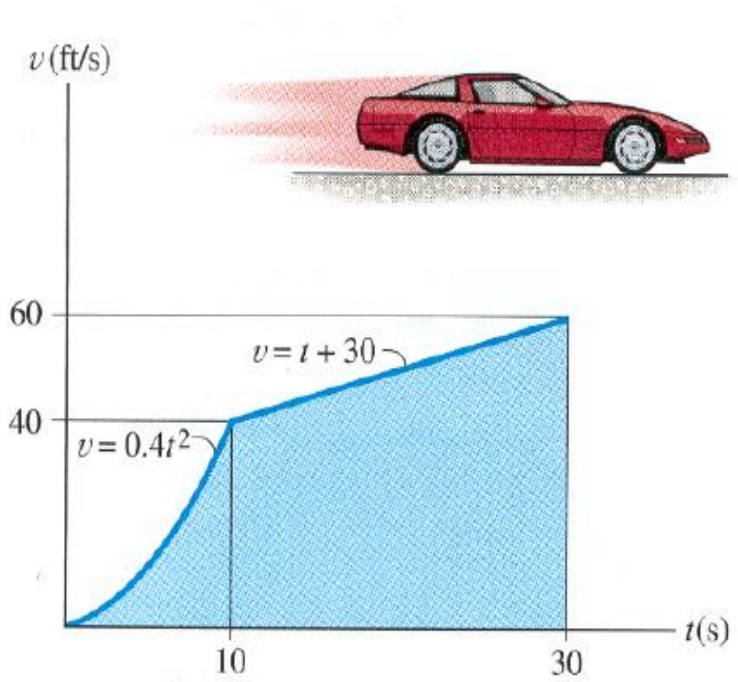
A) 10 s

B) 20 s

C) 30 s

D) 40 s

GROUP PROBLEM SOLVING



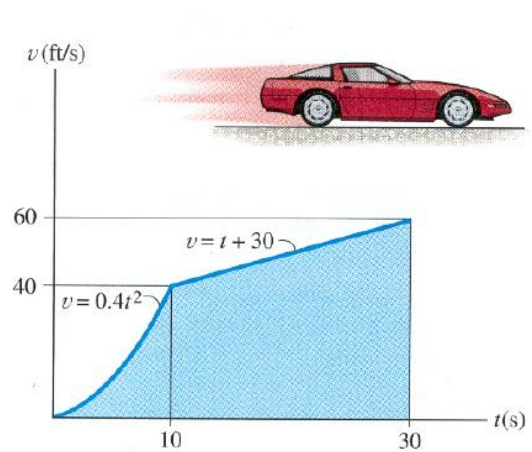
Given: The v-t graph shown

Find: The a-t graph, average speed, and distance traveled for the 30 s interval

Plan: Find slopes of the curves and draw the v-t graph. Find the area under the curve--that is the distance traveled. Finally, calculate average speed (using basic definitions!).

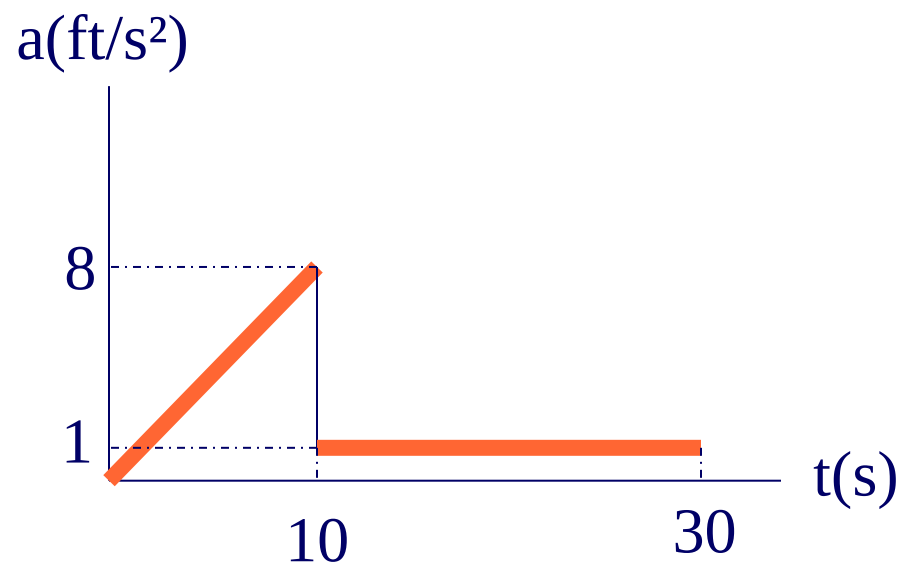
GROUP PROBLEM SOLVING
(continued)

Solution:



For $0 \leq t \leq 10$ $\quad a = dv/dt = 0.8\,t$ ft/s²

For $10 \leq t \leq 30$ $\quad a = dv/dt = 1$ ft/s²



GROUP PROBLEM SOLVING (continued)

$$\Delta s_{0\text{-}10} = \int v\,dt = (1/3)\,(.4)(10)^3 = 400/3 \text{ ft}$$

$$\Delta s_{10\text{-}30} = \int v\,dt = (0.5)(30)^2 + 30(30) - 0.5(10)^2 - 30(10)$$

$$= 1000 \text{ ft}$$

$$s_{0\text{-}30} = 1000 + 400/3 = 1133.3 \text{ ft}$$

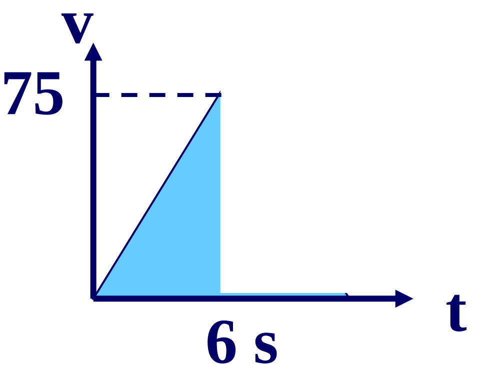
$$v_{avg(0\text{-}30)} = \text{total distance / time}$$

$$= 1133.3/30$$

$$= 37.78 \text{ ft/s}$$

ATTENTION QUIZ

1. If a car has the velocity curve shown, determine the time t necessary for the car to travel 100 meters.



A) 8 s

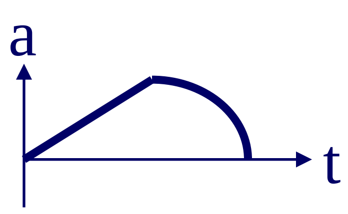
B) 4 s

C) 10 s

D) 6 s

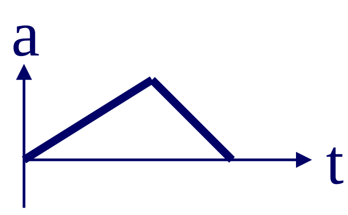
2. Select the correct a-t graph for the velocity curve shown.

A)



B)

C)



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