

# ABSOLUTE DEPENDENT MOTION ANALYSIS OF TWO PARTICLES

## Today's Objectives:

Students will be able to:

1. Relate the positions, velocities, and accelerations of particles undergoing dependent motion.



## In-Class Activities:

- Define Dependent Motion
- Develop Position, Velocity, and Acceleration Relationships



## APPLICATIONS



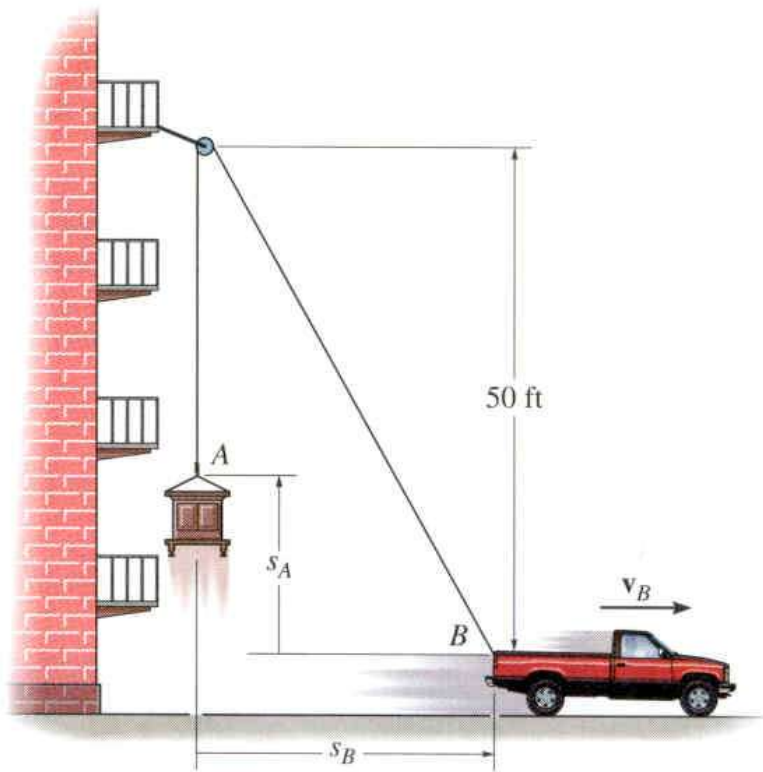
The cable and pulley system shown here can be used to modify the speed of block B relative to the speed of the motor. It is important to relate the various motions in order to determine the power requirements for the motor and the tension in the cable.

If the speed of the cable coming onto the motor pulley is known, how can we determine the speed of block B?



# APPLICATIONS

(continued)



Rope and pulley arrangements are often used to assist in lifting heavy objects. The total lifting force required from the truck depends on the acceleration of the cabinet.

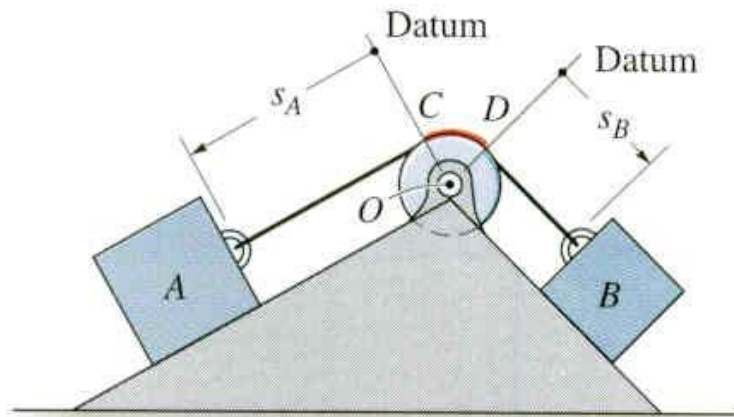
How can we determine the acceleration and velocity of the cabinet if the acceleration of the truck is known?



# DEPENDENT MOTION

## (Section 12.9)

In many kinematics problems, the motion of one object will **depend** on the motion of another object.



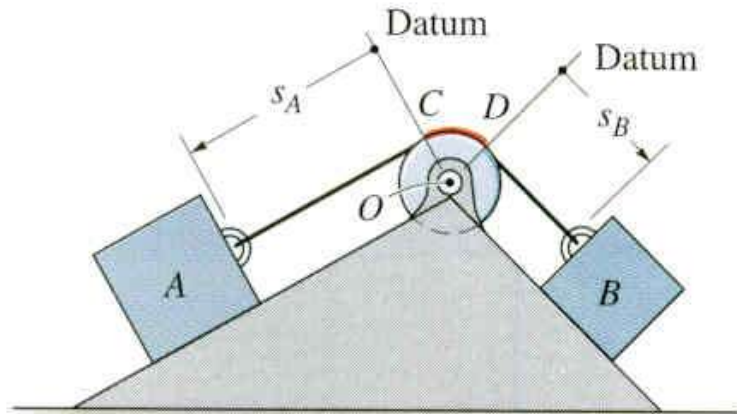
The blocks in this figure are connected by an **inextensible cord** wrapped around a pulley. If block A moves downward along the inclined plane, block B will move up the other incline.

The motion of each block can be related mathematically by defining **position coordinates**,  $s_A$  and  $s_B$ . Each coordinate axis is defined from a **fixed point or datum line**, measured **positive** along each plane in the **direction of motion** of each block.



# DEPENDENT MOTION

(continued)



In this example, position coordinates  $s_A$  and  $s_B$  can be defined from fixed datum lines extending from the center of the pulley along each incline to blocks A and B.

If the cord has a fixed length, the position coordinates  $s_A$  and  $s_B$  are related mathematically by the equation

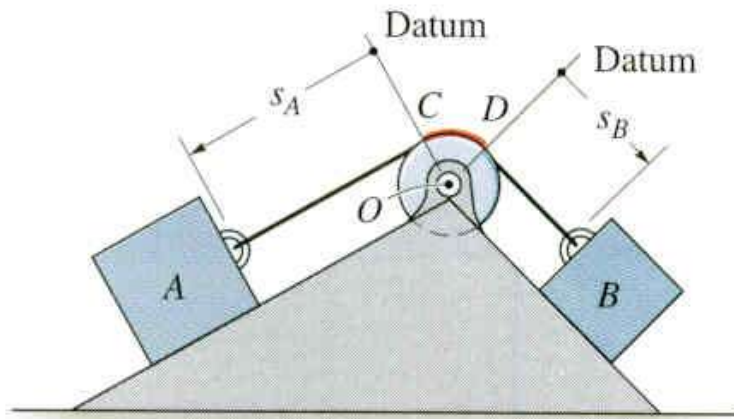
$$s_A + l_{CD} + s_B = l_T$$

Here  $l_T$  is the total cord length and  $l_{CD}$  is the length of cord passing over arc CD on the pulley.



# DEPENDENT MOTION

(continued)



The **velocities** of blocks A and B can be related by **differentiating** the position equation. Note that  **$l_{CD}$  and  $l_T$  remain constant**, so  $dl_{CD}/dt = dl_T/dt = 0$

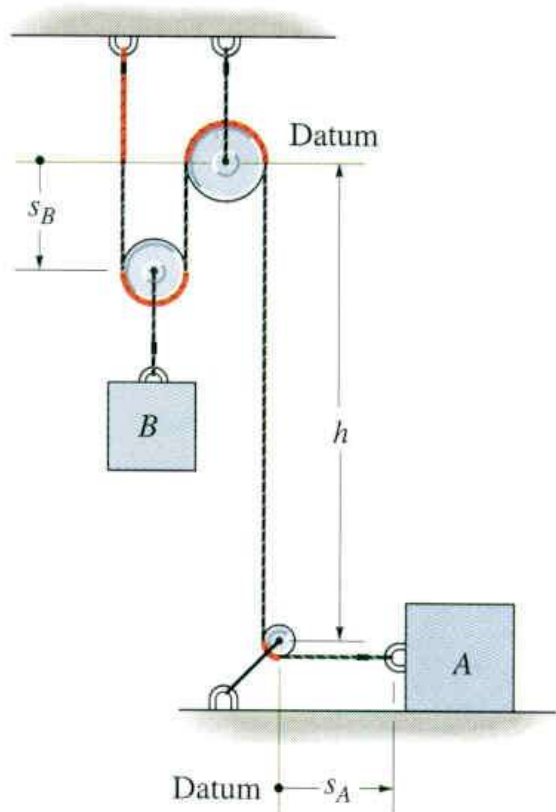
$$ds_A/dt + ds_B/dt = 0 \quad \Rightarrow \quad v_B = -v_A$$

The negative sign indicates that as A moves down the incline (positive  $s_A$  direction), B moves up the incline (negative  $s_B$  direction).

**Accelerations** can be found by **differentiating** the velocity expression. Prove to yourself that  $a_B = -a_A$ .



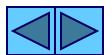
## EXAMPLE



Consider a more complicated example. Position coordinates ( $s_A$  and  $s_B$ ) are defined from fixed datum lines, measured along the direction of motion of each block.

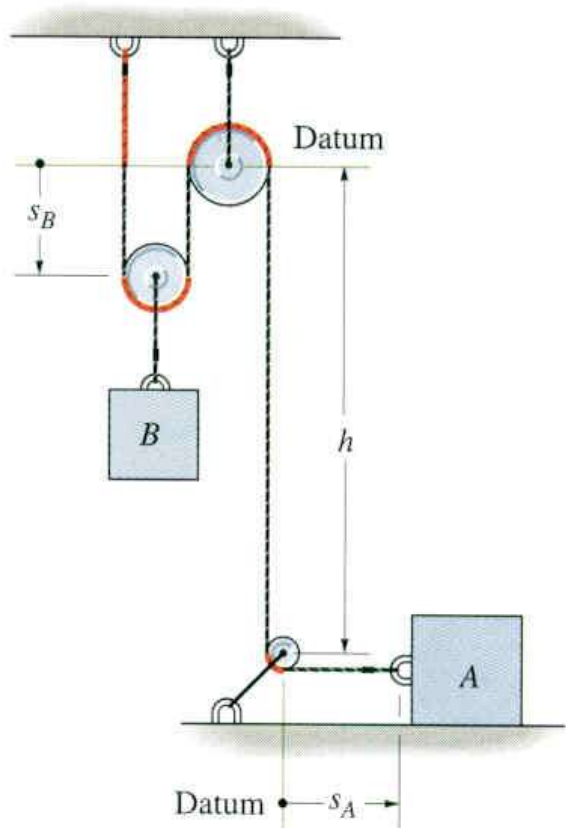
Note that  $s_B$  is only defined to the center of the pulley above block B, since this block moves with the pulley. Also,  $h$  is a constant.

The red colored segments of the cord remain constant in length during motion of the blocks.



# DEPENDENT MOTION EXAMPLE

(continued)



The position coordinates are related by the equation

$$2s_B + h + s_A = l$$

Where  $l$  is the **total cord length minus the lengths of the red segments**.

Since  $l$  and  $h$  remain constant during the motion, the velocities and accelerations can be related by two successive time derivatives:

$$2v_B = -v_A \quad \text{and} \quad 2a_B = -a_A$$

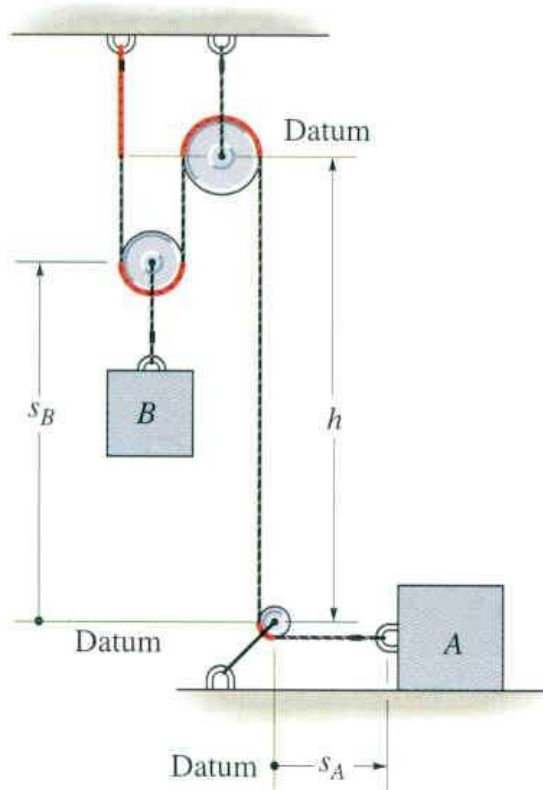
When block B moves downward ( $+s_B$ ), block A moves to the left ( $-s_A$ ). Remember to be **consistent with the sign convention!**





## DEPENDENT MOTION EXAMPLE

(continued)



This example can also be worked by defining the position coordinate for B ( $s_B$ ) from the bottom pulley instead of the top pulley.

The position, velocity, and acceleration relations then become

$$2(h - s_B) + h + s_A = l$$

$$\text{and } 2v_B = v_A \quad 2a_B = a_A$$

Prove to yourself that the **results are the same**, even if the sign conventions are different than the previous formulation.



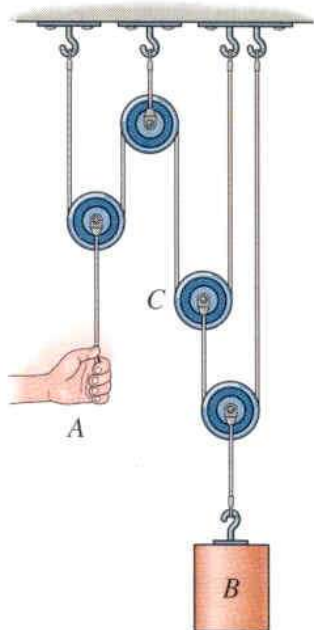
## DEPENDENT MOTION: PROCEDURES

These procedures can be used to relate the **dependent motion** of particles moving along **rectilinear paths** (only the magnitudes of velocity and acceleration change, not their line of direction).

1. Define **position coordinates** from **fixed datum lines**, **along** the **path** of each particle. Different datum lines can be used for each particle.
2. Relate the position coordinates to the cord length. Segments of cord that do **not** change in length during the motion may be **left out**.
3. If a system contains more than one cord, relate the position of a point on one cord to a point on another cord. **Separate equations are written for each cord**.
4. **Differentiate** the position coordinate equation(s) to relate **velocities** and **accelerations**. Keep track of signs!



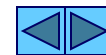
## EXAMPLE II



**Given:** In the figure on the left, the cord at A is pulled down with a speed of 8 ft/s.

**Find:** The speed of block B.

**Plan:** There are two cords involved in the motion in this example. *The position of a point on one cord must be related to the position of a point on the other cord.* There will be two position equations (one for each cord).

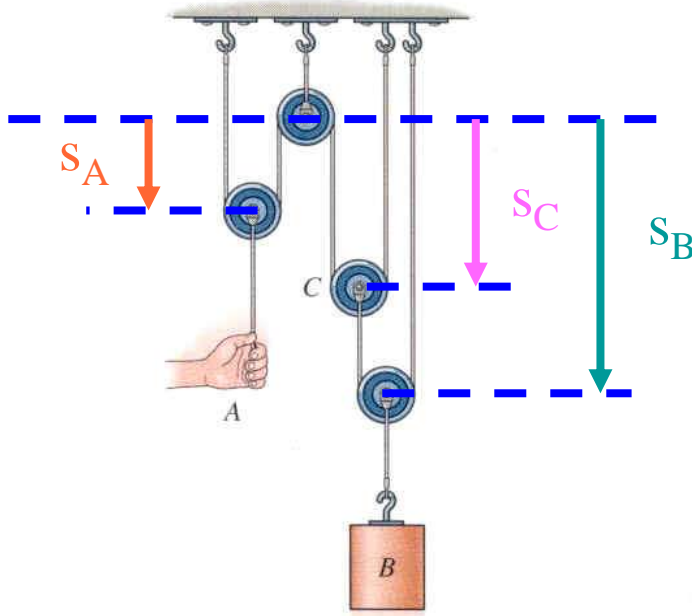


## EXAMPLE

(continued)

### Solution:

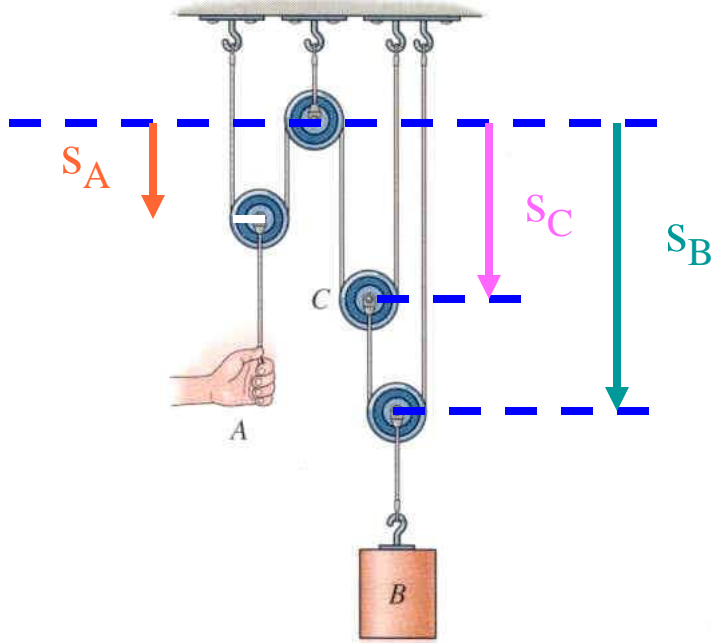
- 1) Define the position coordinates from a fixed datum line. Three coordinates must be defined: one for point A ( $s_A$ ), one for block B ( $s_B$ ), and one relating positions on the two cords. Note that pulley C relates the motion of the two cords.



- Define the datum line through the top pulley (which has a fixed position).
- $s_A$  can be defined to the center of the pulley above point A.
- $s_B$  can be defined to the center of the pulley above B.
- $s_C$  is defined to the center of pulley C.
- All coordinates are defined as positive down and along the direction of motion of each point/object. ◀▶

## EXAMPLE

(continued)



- 2) Write position/length equations for each cord. Define  $l_1$  as the length of the first cord, minus any segments of constant length. Define  $l_2$  in a similar manner for the second cord:

$$\text{Cord 1: } 2s_A + 2s_C = l_1$$

$$\text{Cord 2: } s_B + (s_B - s_C) = l_2$$

- 3) Eliminating  $s_C$  between the two equations, we get

$$2s_A + 4s_B = l_1 + 2l_2$$

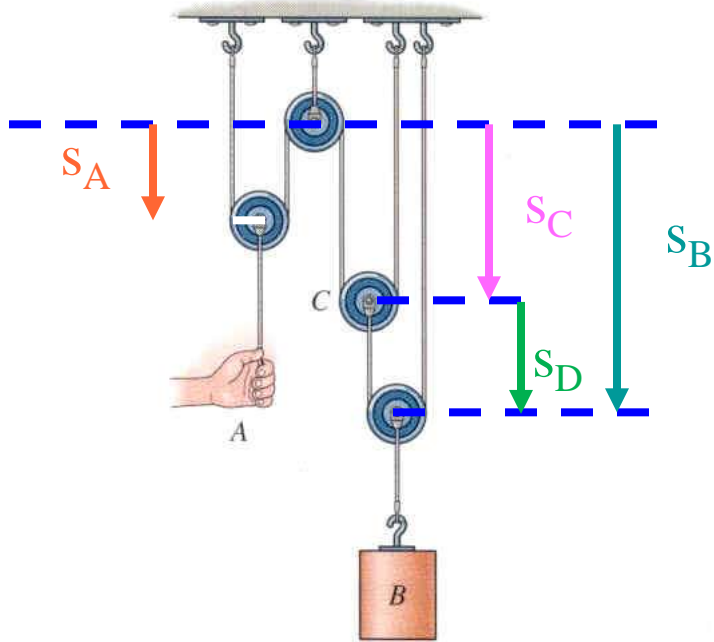
- 4) Relate velocities by differentiating this expression. Note that  $l_1$  and  $l_2$  are constant lengths.

$$2v_A + 4v_B = 0 \quad \Rightarrow \quad v_B = -0.5v_A = -0.5(8) = -4 \text{ ft/s}$$

The velocity of block B is 4 ft/s up (negative  $s_B$  direction).



## EXAMPLE (continued)



Be careful using 2 moving points,  
i.e.  $s_D$ .

$$\text{Cord 1: } 2s_A + 2s_C = l_1$$

$$\text{Cord 2: } s_B + s_D = l_2$$

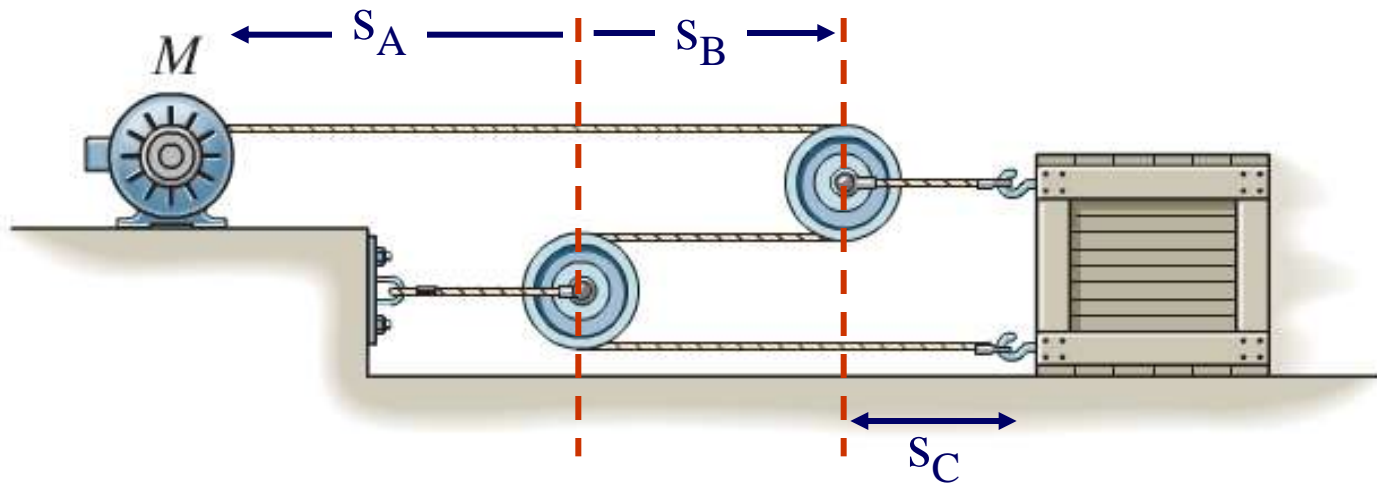
No common term in two equations.

Need a third equation

$$s_C + s_D = s_B$$



# Cable of Varying Length

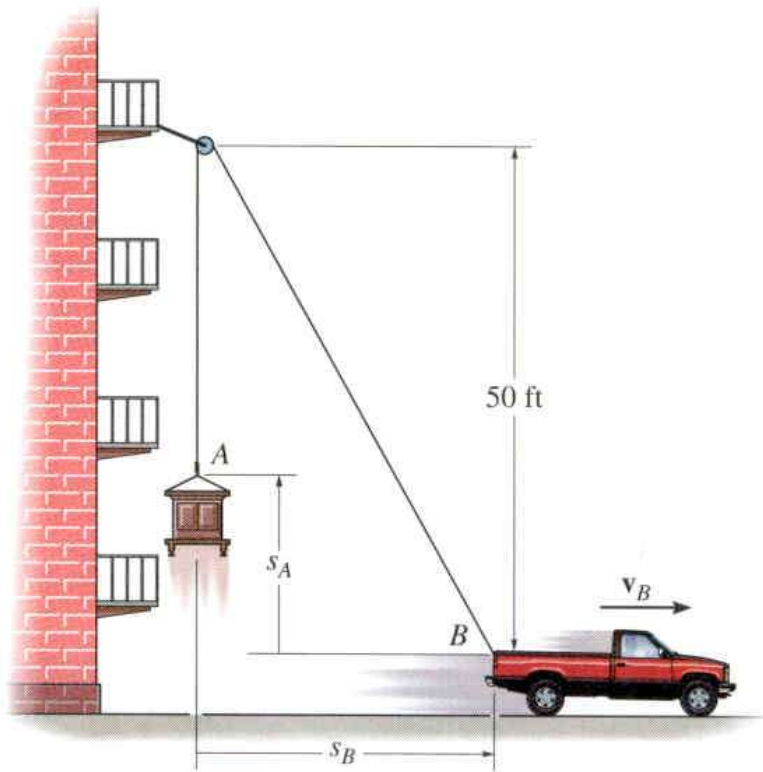


$$s_A + 3s_B + s_C = L$$

$$0 + 3v_B + 0 = v_M$$

Both  $v_M$  are  $v_B$  negative since they get smaller.

## Example III (2D)



**Given:** In the figure on the left, the truck has  $v_B = 5 \text{ m/s}$  and  $a_B = 1 \text{ m/s}^2$  when  $s_B = 15 \text{ ft}$ .

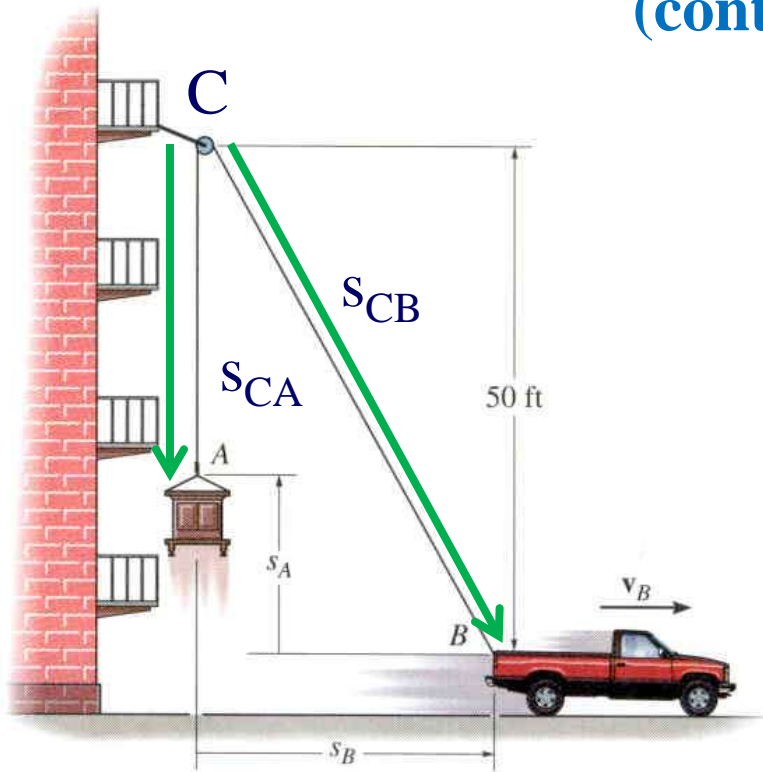
**Find:** The speed and acceleration of cabinet

**Plan:** There is two cords involved.  
Use Pythagoras.  
Differentiation is easier with squares than square roots.





## Example III (2D) (continued)



$$1: s_{CA} + s_{CB} = L$$

$$2: s_{CA} + s_A = 50$$

$$3: s_B^2 + 50^2 = s_{CB}^2$$

$$1': v_{CA} + v_{CB} = 0,$$

$$a_{CA} + a_{CB} = 0$$

$$2': v_{CA} + v_A = 0,$$

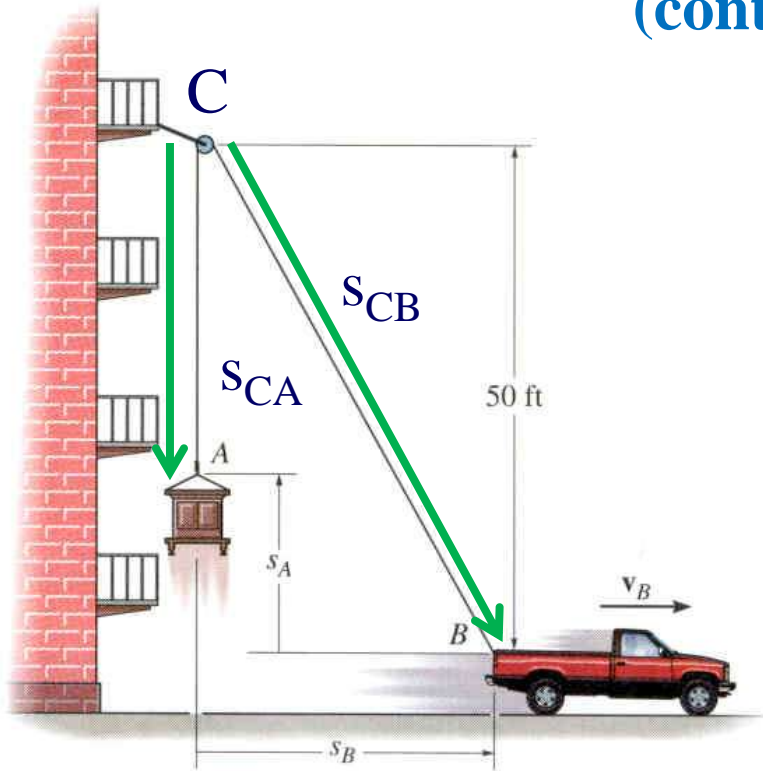
$$a_{CA} + a_A = 0$$

$$3': 2s_B v_B = 2s_{CB} v_{CB}$$

$$v_B^2 + s_B a_B = v_{CB}^2 + s_{CB} a_B$$



## Example III (2D) (continued)



From 1' and 2',  $v_{CB} = v_A$  and  
 $a_{CB} = a_A$ .

From 2,  $s_{CB}^2 = 15^2 + 50^2$ ,  
 $s_{CB} = 52.20$  m.

Using 3',

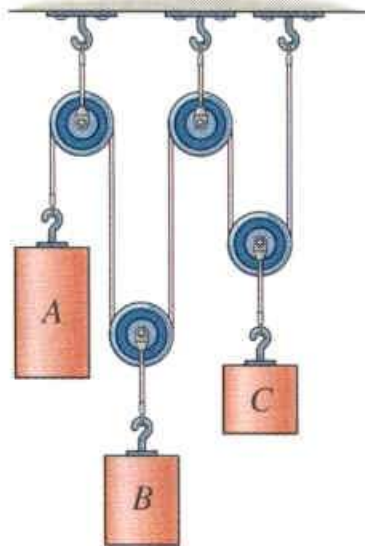
$$v_A = s_B v_B / s_{CB} = 15 * 5 / 42.20 \\ = 1.437 \text{ m/s}$$

3' becomes  $v_B^2 + s_B a_B = v_A^2 + s_{CB} a_A$ , so

$$a_A = (v_B^2 + s_B a_B - v_A^2) / s_{CB} = (5^2 + 15 * 1 - 1.437^2) / 52.20 \\ = 0.7267 \text{ m/s}^2$$



## GROUP PROBLEM SOLVING



**Given:** In this pulley system, block A is moving downward with a speed of 4 ft/s while block C is moving up at 2 ft/s.

**Find:** The speed of block B.

**Plan:** All blocks are connected to a single cable, so only one position/length equation will be required. Define position coordinates for each block, write out the position relation, and then differentiate it to relate the velocities.

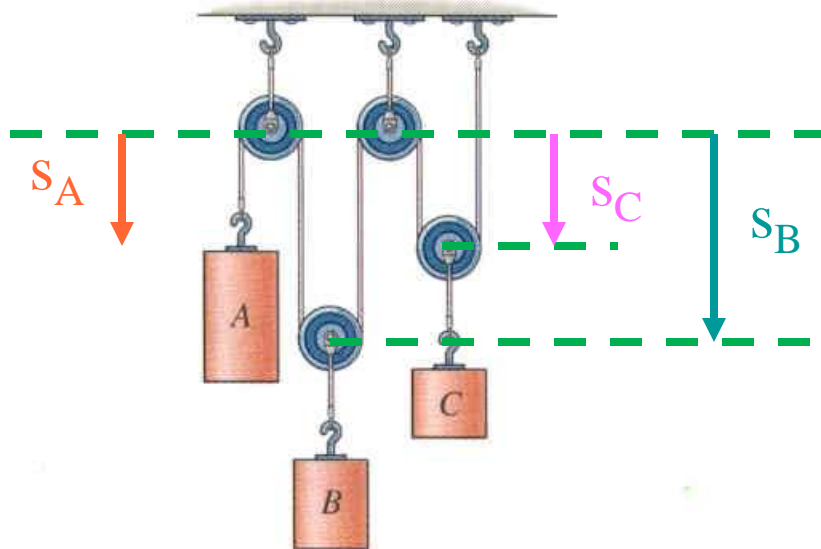


# GROUP PROBLEM SOLVING

(continued)

**Solution:**

- 1) A datum line can be drawn through the upper, fixed, pulleys and position coordinates defined from this line to each block (or the pulley above the block).



- 2) Defining  $s_A$ ,  $s_B$ , and  $s_C$  as shown, the position relation can be written:

$$s_A + 2s_B + 2s_C = 1$$

- 3) Differentiate to relate velocities:

$$v_A + 2v_B + 2v_C = 0$$

$$\Rightarrow 4 + 2v_B + 2(-2) = 0$$

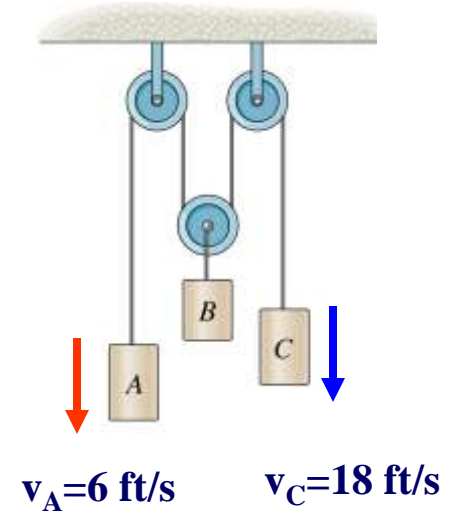
$$\Rightarrow v_B = 0$$



## ATTENTION QUIZ

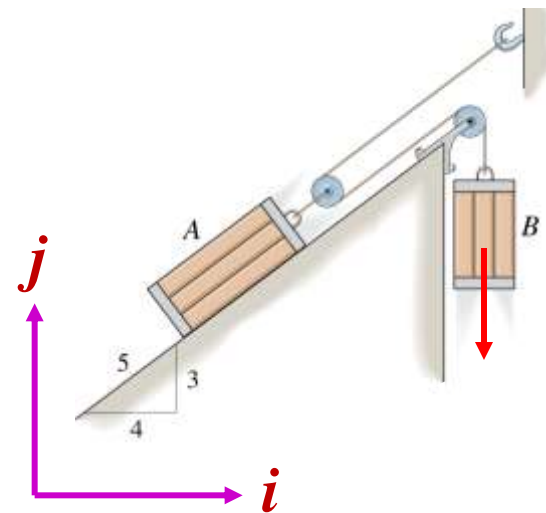
1. Determine the speed of block B when block A is moving down at 6 ft/s while block C is moving down at 18 ft/s .

- A) 24 ft/s                      B) 3 ft/s  
C) 12 ft/s                      D) 9 ft/s

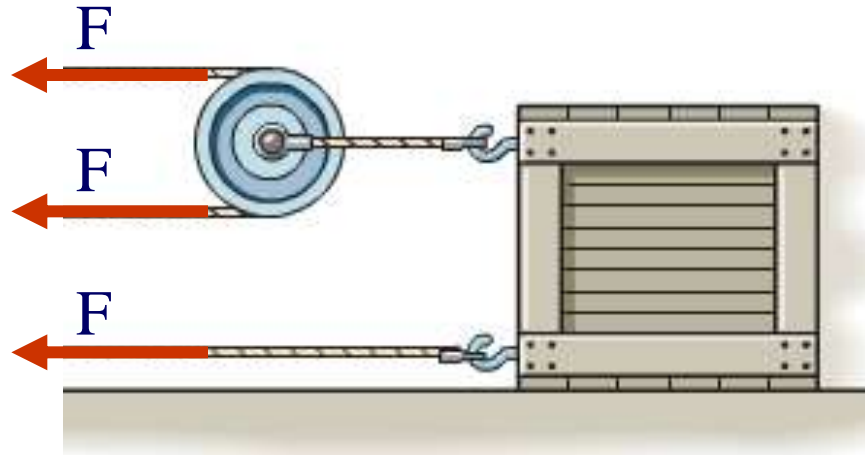
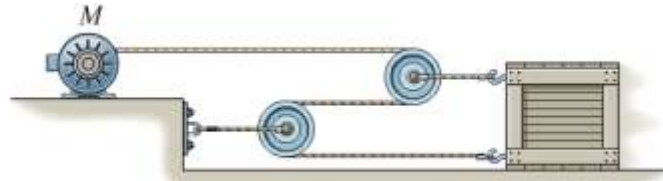


2. Determine the velocity vector of block A when block B is moving downward with a speed of 10 m/s.

- A)  $(8\mathbf{i} + 6\mathbf{j}) \text{ m/s}$     B)  $(4\mathbf{i} + 3\mathbf{j}) \text{ m/s}$   
C)  $(-8\mathbf{i} - 6\mathbf{j}) \text{ m/s}$     D)  $(3\mathbf{i} + 4\mathbf{j}) \text{ m/s}$



# Pulleys are Force Multipliers



$$3F = Ma$$