

# CURVILINEAR MOTION: GENERAL & RECTANGULAR COMPONENTS

## Today's Objectives:

Students will be able to:

1. Describe the motion of a particle traveling along a curved path.
2. Relate kinematic quantities in terms of the rectangular components of the vectors.



## In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- General Curvilinear Motion
- Rectangular Components of Kinematic Vectors
- Concept Quiz
- Group Problem Solving
- Attention Quiz



## READING QUIZ

1. In curvilinear motion, the direction of the instantaneous velocity is always
  - A) tangent to the hodograph.
  - B) perpendicular to the hodograph.
  - C) tangent to the path.
  - D) perpendicular to the path.
  
2. In curvilinear motion, the direction of the instantaneous acceleration is always
  - A) tangent to the hodograph.
  - B) perpendicular to the hodograph.
  - C) tangent to the path.
  - D) perpendicular to the path.



## APPLICATIONS



The path of motion of each plane in this formation can be tracked with radar and their  $x$ ,  $y$ , and  $z$  coordinates (relative to a point on earth) recorded as a function of time.

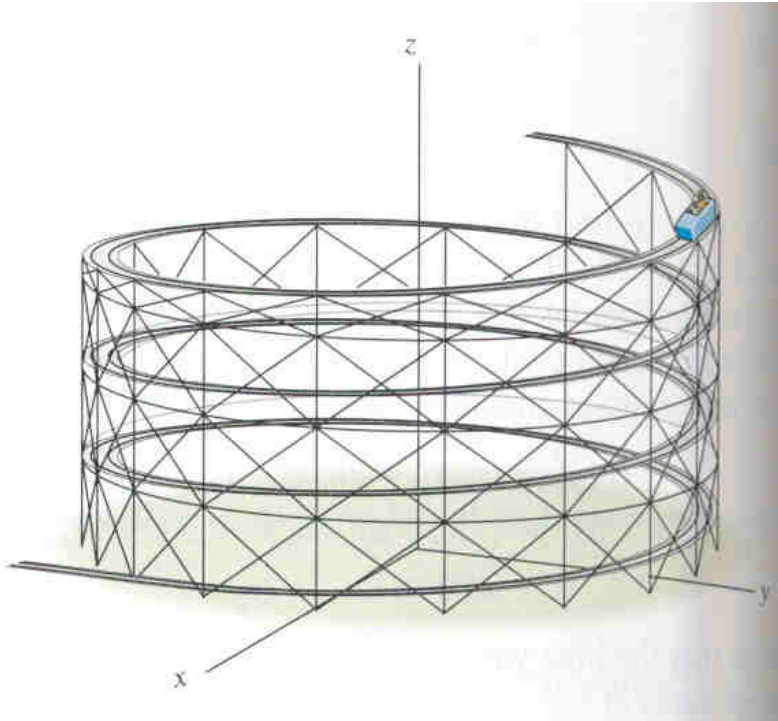
How can we determine the velocity or acceleration of each plane at any instant?

Should they be the same for each aircraft?



# APPLICATIONS

(continued)



A roller coaster car travels down a fixed, helical path at a constant speed.

How can we determine its position or acceleration at any instant?

If you are designing the track, why is it important to be able to predict the acceleration of the car?

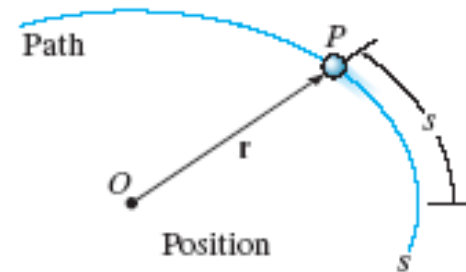


# GENERAL CURVILINEAR MOTION

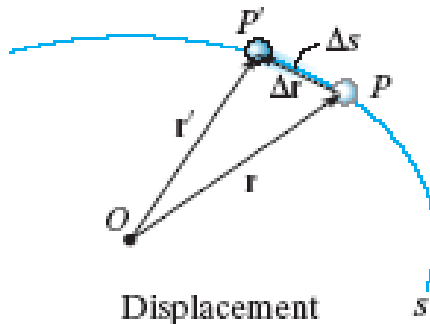
## (Section 12.4)

A particle moving along a curved path undergoes **curvilinear motion**. Since the motion is often three-dimensional, **vectors** are used to describe the motion.

A particle moves along a curve defined by the path function,  $s$ .



The **position** of the particle at any instant is designated by the vector  $\mathbf{r} = \mathbf{r}(t)$ . Both the **magnitude** and **direction** of  $\mathbf{r}$  may vary with time.

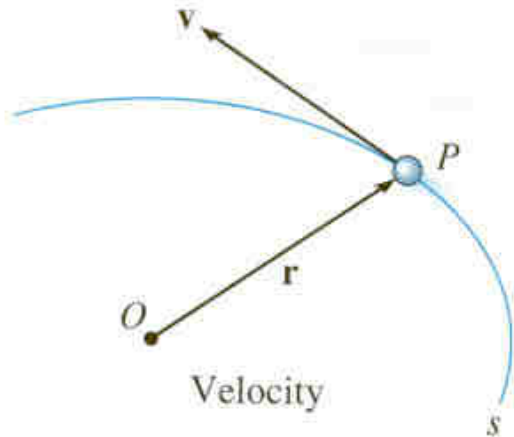


If the particle moves a distance  $\Delta s$  along the curve during time interval  $\Delta t$ , the **displacement** is determined by **vector subtraction**:  $\Delta \mathbf{r} = \mathbf{r}' - \mathbf{r}$



# VELOCITY

**Velocity** represents the rate of change in the position of a particle.



The **average velocity** of the particle during the time increment  $\Delta t$  is

$$\mathbf{v}_{avg} = \Delta \mathbf{r} / \Delta t .$$

The **instantaneous velocity** is the time-derivative of position

$$\mathbf{v} = d\mathbf{r} / dt .$$

The **velocity vector**,  $\mathbf{v}$ , is **always** tangent to the path of motion.

The magnitude of  $\mathbf{v}$  is called the **speed**. Since the arc length  $\Delta s$  approaches the magnitude of  $\Delta \mathbf{r}$  as  $t \rightarrow 0$ , the speed can be obtained by differentiating the path function ( $v = ds/dt$ ). Note that this is not a vector!



# ACCELERATION

**Acceleration** represents the rate of change in the velocity of a particle.

If a particle's velocity changes from  $\mathbf{v}$  to  $\mathbf{v}'$  over a time increment  $\Delta t$ , the **average acceleration** during that increment is:

$$\mathbf{a}_{avg} = \Delta \mathbf{v} / \Delta t = (\mathbf{v} - \mathbf{v}') / \Delta t$$

The **instantaneous acceleration** is the time-derivative of velocity:

$$\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2$$

A plot of the locus of points defined by the arrowhead of the velocity vector is called a **hodograph**. The acceleration vector is tangent to the hodograph, but not, in general, tangent to the path function.

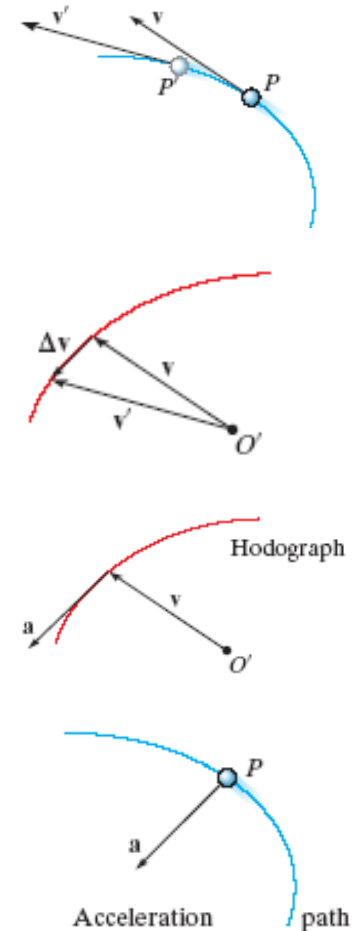


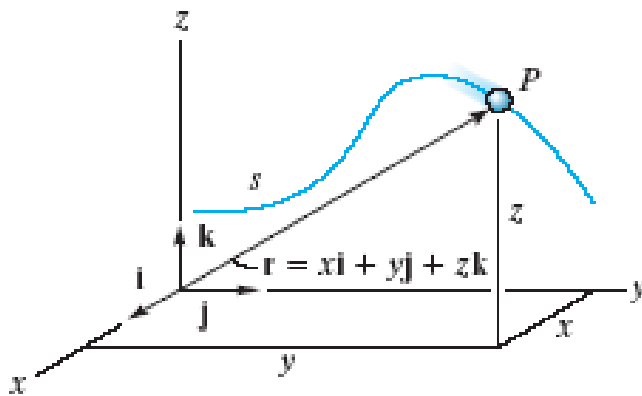
Fig. 12-16



# CURVILINEAR MOTION: RECTANGULAR COMPONENTS

## (Section 12.5)

It is often convenient to describe the motion of a particle in terms of its  $x$ ,  $y$ ,  $z$  or **rectangular components**, relative to a **fixed frame of reference**.



Position

The position of the particle can be defined at any instant by the **position vector**

$$\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k} .$$

The  $x$ ,  $y$ ,  $z$  components may all be **functions of time**, i.e.,

$$x = x(t), y = y(t), \text{ and } z = z(t) .$$

The **magnitude** of the position vector is:  $r = (x^2 + y^2 + z^2)^{0.5}$

The **direction** of  $\mathbf{r}$  is defined by the unit vector:  $\mathbf{u}_r = (1/r)\mathbf{r}$





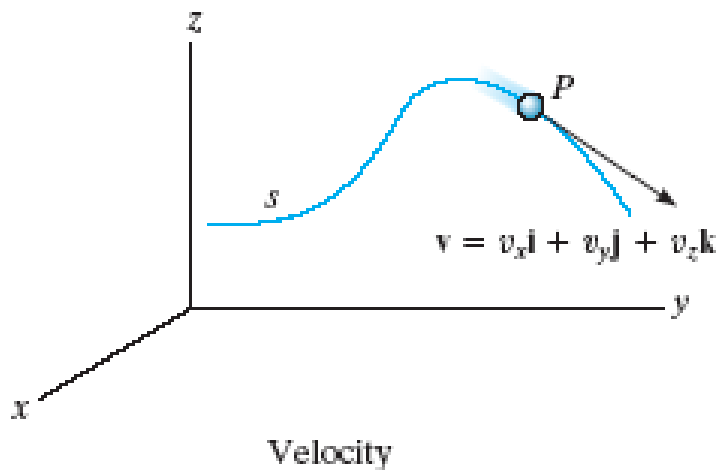
## RECTANGULAR COMPONENTS: VELOCITY

The **velocity vector** is the time derivative of the position vector:

$$\mathbf{v} = d\mathbf{r}/dt = d(x\mathbf{i})/dt + d(y\mathbf{j})/dt + d(z\mathbf{k})/dt$$

Since the **unit vectors  $\mathbf{i}, \mathbf{j}, \mathbf{k}$**  are **constant in magnitude and direction**, this equation reduces to  $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$

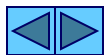
where  $v_x = \dot{x} = dx/dt$ ,  $v_y = \dot{y} = dy/dt$ ,  $v_z = \dot{z} = dz/dt$



The **magnitude** of the velocity vector is

$$v = [(v_x)^2 + (v_y)^2 + (v_z)^2]^{0.5}$$

The **direction** of  $\mathbf{v}$  is **tangent** to the path of motion.



# RECTANGULAR COMPONENTS: ACCELERATION

The **acceleration vector** is the time derivative of the velocity vector (second derivative of the position vector):

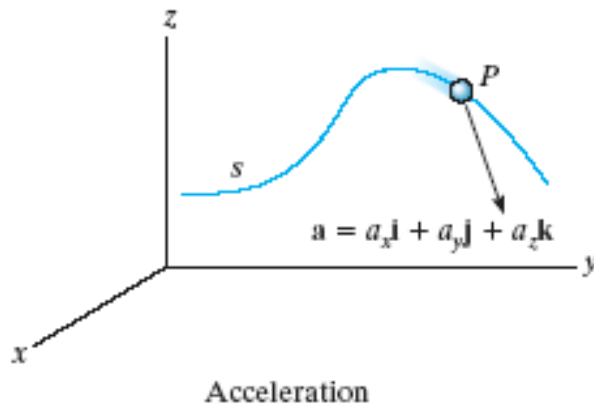
$$\mathbf{a} = d\mathbf{v}/dt = d^2\mathbf{r}/dt^2 = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

where  $a_x = \dot{v}_x = \ddot{x} = dv_x/dt$ ,  $a_y = \dot{v}_y = \ddot{y} = dv_y/dt$ ,

$a_z = \dot{v}_z = \ddot{z} = dv_z/dt$

The **magnitude** of the acceleration vector is

$$a = [(a_x)^2 + (a_y)^2 + (a_z)^2]^{0.5}$$



The **direction** of  $\mathbf{a}$  is usually **not tangent** to the path of the particle.



## EXAMPLE

**Given:** The motion of two particles (A and B) is described by the position vectors

$$\mathbf{r}_A = [3t \mathbf{i} + 9t(2 - t) \mathbf{j}] \text{ m}$$

$$\mathbf{r}_B = [3(t^2 - 2t + 2) \mathbf{i} + 3(t - 2) \mathbf{j}] \text{ m}$$

**Find:** The point at which the particles collide and their speeds just before the collision.

- Plan:**
- 1) The particles will collide when their position vectors are equal, or  $\mathbf{r}_A = \mathbf{r}_B$ .
  - 2) Their speeds can be determined by differentiating the position vectors.



## EXAMPLE

(continued)

### Solution:

- 1) The point of collision requires that  $\mathbf{r}_A = \mathbf{r}_B$ , so  $x_A = x_B$  and  $y_A = y_B$ .

$$\text{x-components: } 3t = 3(t^2 - 2t + 2)$$

$$\text{Simplifying: } t^2 - 3t + 2 = 0$$

$$\text{Solving: } t = \{3 \pm [3^2 - 4(1)(2)]^{0.5}\} / 2(1)$$

$$\Rightarrow t = 2 \text{ or } 1 \text{ s}$$

$$\text{y-components: } 9t(2 - t) = 3(t - 2)$$

$$\text{Simplifying: } 3t^2 - 5t - 2 = 0$$

$$\text{Solving: } t = \{5 \pm [5^2 - 4(3)(-2)]^{0.5}\} / 2(3)$$

$$\Rightarrow t = 2 \text{ or } -1/3 \text{ s}$$

So, the particles collide when  $t = 2$  s. Substituting this value into  $\mathbf{r}_A$  or  $\mathbf{r}_B$  yields

$$x_A = x_B = 6 \text{ m} \quad \text{and} \quad y_A = y_B = 0$$



## EXAMPLE

(continued)

2) Differentiate  $\mathbf{r}_A$  and  $\mathbf{r}_B$  to get the velocity vectors.

$$\mathbf{v}_A = d\mathbf{r}_A/dt = \dot{x}_A \mathbf{i} + \dot{y}_A \mathbf{j} = [3\mathbf{i} + (18 - 18t)\mathbf{j}] \text{ m/s}$$

$$\text{At } t = 2 \text{ s: } \mathbf{v}_A = [3\mathbf{i} - 18\mathbf{j}] \text{ m/s}$$

$$\mathbf{v}_B = d\mathbf{r}_B/dt = \dot{x}_B \mathbf{i} + \dot{y}_B \mathbf{j} = [(6t - 6)\mathbf{i} + 3\mathbf{j}] \text{ m/s}$$

$$\text{At } t = 2 \text{ s: } \mathbf{v}_B = [6\mathbf{i} + 3\mathbf{j}] \text{ m/s}$$

Speed is the magnitude of the velocity vector.

$$v_A = (3^2 + 18^2)^{0.5} = 18.2 \text{ m/s}$$

$$v_B = (6^2 + 3^2)^{0.5} = 6.71 \text{ m/s}$$



## CONCEPT QUIZ

1. If the position of a particle is defined by  $\mathbf{r} = [(1.5t^2 + 1) \mathbf{i} + (4t - 1) \mathbf{j}]$  (m), its speed at  $t = 1$  s is  
A) 2 m/s  
B) 3 m/s  
C) 5 m/s  
D) 7 m/s
2. The path of a particle is defined by  $y = 0.5x^2$ . If the component of its velocity along the x-axis at  $x = 2$  m is  $v_x = 1$  m/s, its velocity component along the y-axis at this position is  
A) 0.25 m/s  
B) 0.5 m/s  
C) 1 m/s  
D) 2 m/s



## GROUP PROBLEM SOLVING

**Given:** A particle travels along a path described by the parabola  $y = 0.5x^2$ . The x-component of velocity is given by  $v_x = (5t)$  ft/s. When  $t = 0$ ,  $x = y = 0$ .

**Find:** The particle's distance from the origin and the magnitude of its acceleration when  $t = 1$  s.

**Plan:** Note that  $v_x$  is given as a function of time.

- 1) Determine the x-component of position and acceleration by integrating and differentiating  $v_x$ , respectively.
- 2) Determine the y-component of position from the parabolic equation and differentiate to get  $a_y$ .
- 3) Determine the magnitudes of the position and acceleration vectors.



# GROUP PROBLEM SOLVING

(continued)

## Solution:

### 1) x-components:

Velocity:  $v_x = \dot{x} = dx/dt = (5t) \text{ ft/s}$

Position:  $\int_0^x dx = \int_0^t 5t dt \Rightarrow x = (5/2)t^2 = (2.5t^2) \text{ ft}$

Acceleration:  $a_x = \ddot{x} = \dot{v}_x = d/dt (5t) = 5 \text{ ft/s}^2$

### 2) y-components:

Position:  $y = 0.5x^2 = 0.5(2.5t^2)^2 = (3.125t^4) \text{ ft}$

Velocity:  $v_y = dy/dt = d(3.125t^4)/dt = (12.5t^3) \text{ ft/s}$

Acceleration:  $a_y = \dot{v}_y = d(12.5t^3)/dt = (37.5t^2) \text{ ft/s}^2$





## GROUP PROBLEM SOLVING

(continued)

- 3) The **distance** from the origin is the **magnitude** of the position vector:

$$\mathbf{r} = x \mathbf{i} + y \mathbf{j} = [2.5t^2 \mathbf{i} + 3.125t^4 \mathbf{j}] \text{ ft}$$

$$\text{At } t = 1 \text{ s, } \mathbf{r} = (2.5 \mathbf{i} + 3.125 \mathbf{j}) \text{ ft}$$

$$\text{Distance: } d = r = (2.5^2 + 3.125^2)^{0.5} = 4.0 \text{ ft}$$

The **magnitude** of the acceleration vector is calculated as:

$$\text{Acceleration vector: } \mathbf{a} = [5 \mathbf{i} + 37.5t^2 \mathbf{j}] \text{ ft/s}^2$$

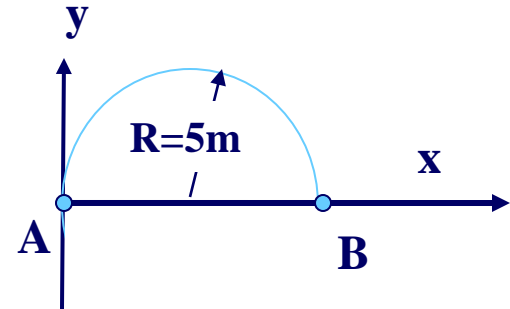
$$\text{Magnitude: } a = (5^2 + 37.5^2)^{0.5} = 37.8 \text{ ft/s}^2$$



## ATTENTION QUIZ

1. If a particle has moved from A to B along the circular path in 4s, what is the average velocity of the particle ?

- A)  $2.5 \mathbf{i}$  m/s
- B)  $2.5 \mathbf{i} + 1.25 \mathbf{j}$  m/s
- C)  $1.25 \pi \mathbf{i}$  m/s
- D)  $1.25 \pi \mathbf{j}$  m/s



2. The position of a particle is given as  $\mathbf{r} = (4t^2 \mathbf{i} - 2x \mathbf{j})$  m. Determine the particle's acceleration.

- A)  $(4 \mathbf{i} + 8 \mathbf{j})$  m/s<sup>2</sup>
- B)  $(8 \mathbf{i} - 16 \mathbf{j})$  m/s<sup>2</sup>
- C)  $(8 \mathbf{i})$  m/s<sup>2</sup>
- D)  $(8 \mathbf{j})$  m/s<sup>2</sup>

