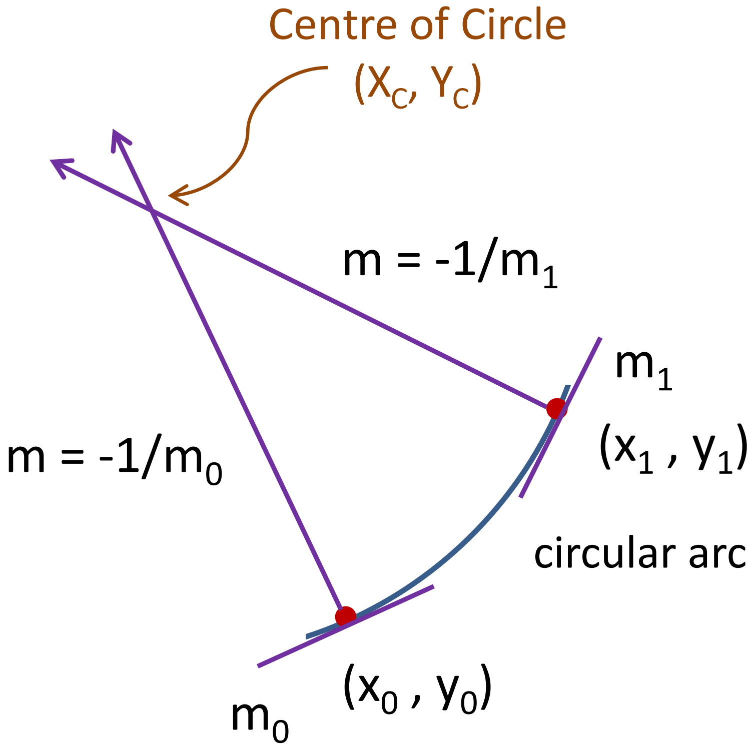


Can we find ρ in terms of knowns m_0, m_1, x_0 and y_0 ?

Since we have the slope of each line and a point they pass through, we can get the equation of each y = mx +b.

Line 0: slope $m = -1/m_0$, and to find b use from point
$y_0 = -(1/m_0)x_0 + b$

$\therefore y = -(1/m_0)x + (y_0 + (1/m_0)x_0)$

or $y - y_0 = -(1/m_0)(x - x_0)$ [Eqn A]

Line 1: slope $m = -1/m_1$, and to find b use from point
$y_1 = -(1/m_1)x_1 + b$

$\therefore y = -(1/m_1)x + (y_1 + (1/m_1)x_1)$

or $y - y_1 = -(1/m_1)(x - x_1)$

When $x = X_C$ in each equation, $y = Y_C$. So

$$-(1/m_0)X_C + (y_0 + (1/m_0)x_0) = -(1/m_1)X_C + (y_1 + (1/m_1)x_1)$$

Collecting X_C terms

$$[(1/m_1) – (1/m_0)]X_C = y_1 – y_0 + (1/m_1)x_1 – (1/m_0)x_0$$

Rearranging into a more useful form for later:

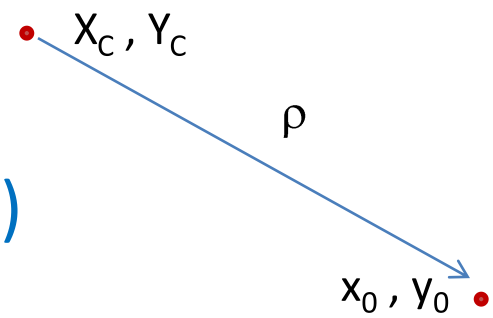
$$[(1/m_1) – (1/m_0)](X_C – x_0) = y_1 – y_0 + (1/m_1)x_1 – (1/m_0)x_0 – [(1/m_1) – (1/m_0)]x_0$$

Or

$$[(1/m_1) – (1/m_0)](X_C – x_0) = y_1 – y_0 + (1/m_1)(x_1 – x_0) \quad \text{[Eqn B]}$$

Now that we have found an Equation for X_C we can determine ρ using Pythagorean theorem

$$(X_C - x_0)^2 + (Y_C - y_0)^2 = \rho^2$$



From Eqn A , $Y_C - y_0 = -(1/m_0)(X_C - x_0)$

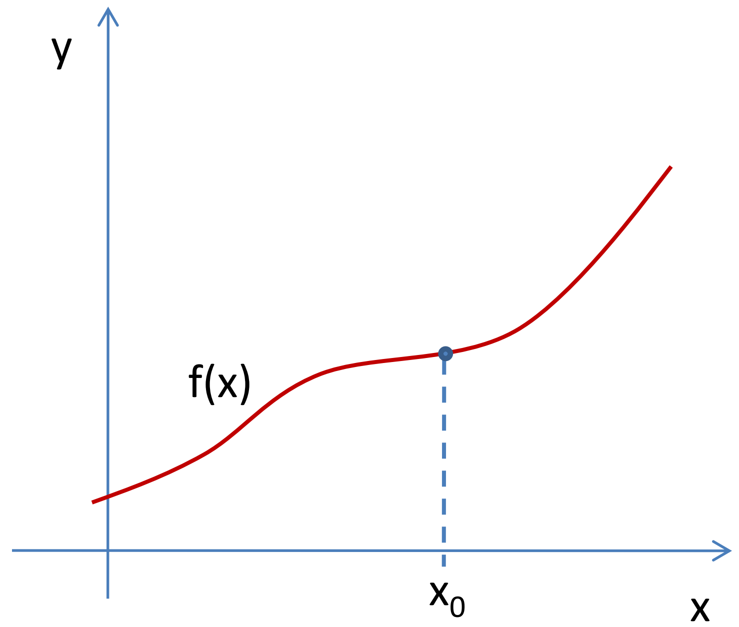
$$(1 + 1/m_0^2)(X_C - x_0)^2 = \rho^2$$

We can rewrite Eqn B

$$(X_C - x_0) = \{y_1 - y_0 + (1/m_1)(x_1 - x_0)\} * m_1 m_0 / (m_1 - m_0)$$

$$(1 + 1/m_0^2) \{y_1 - y_0 + (1/m_1)(x_1 - x_0)\}^2 [m_1 m_0 / (m_1 - m_0)]^2 = \rho^2$$

Find radius of curve at $x = x_0$ where $y = f(x)$ is given.



Hold on! We do have x_0, $y_0 = f(x_0)$, and $m_0 = f'(x_0)$, but we don't have x_1, $y_1 = f(x_1)$, and $m_1 = f'(x_1)$. And this is certainly not a circular arc!

Well if we consider a point dx to the right, the portion of the curve from x_0 to x_0 + dx is a very good approximation of a circular arc.

Thus $x_1 = x_0 + dx$, $y_1 = f(x_0 + dx)$ and $m_1 = f'(x_0 + dx)$.

Furthermore, using a Taylor expansion

$$y_1 = f(x_0 + dx) = f(x_0) + dx\, f'(x_0) = y_0 + m_0\, dx \text{ , and}$$

$$m_1 = f'(x_0 + dx) = f'(x_0) + dx\, f''(x_0) = m_0 + dx\, f''(x_0)$$

Since we know f(x), we do know how to get first and second derivatives.

$(1 + 1/m_0^2)\ \{y_1 - y_0 + (1/m_1)(x_1 - x_0)\}^2\ [m_1 m_0 / (m_1 - m_0)]^2 = \rho^2$

$(1 + 1/m_0^2)\{m_0 dx + dx/(\ m_0 + dx\ f''(x_0)\ )\}^2$
$*[(\ m_0 + dx\ f''(x_0)\ )\ m_0\ ]^2 / \ [dx\ f''(x_0)\ ]^2 = \rho^2$

Since dx is vanishingly small, can expand

$(m_0^2 + 1)/m_0^2\ \{m_0 dx + (dx/m_0)(1 + dx\ f''(x_0)\ /m_0)\}^2$
$*[(\ 1 + dx\ f''(x_0)/m_0\ )\ m_0^2]^2 / \ [dx\ f''(x_0)\ ]^2 = \rho^2$

$(m_0^2 + 1)/m_0^2\ (m_0 dx)^2\ \{\ 1 + 1/m_0^2\ (1 + dx\ f''(x_0)\ /m_0)\}^2$
$*[(\ 1 + dx\ f''(x_0)/m_0\ )\ ]^2\ m_0^4 / \ [dx\ f''(x_0)\ ]^2 = \rho^2$

$$(m_0^2 + 1) \{ 1 + 1/m_0^2 (1 + dx\, f''(x_0) / m_0)\}^2$$
$$* [(1 + dx\, f''(x_0)/m_0)]^2 m_0^4 / [f''(x_0)]^2 = \rho^2$$

Letting dx vanish we have:

$$(m_0^2 + 1) \{ 1 + 1/m_0^2 \}^2 m_0^4 / [f''(x_0)]^2 = \rho^2$$

$$(m_0^2 + 1)^3 / [f''(x_0)]^2 = \rho^2$$

$$([f'(x_0)]^2 + 1)^3 / [f''(x_0)]^2 = \rho^2$$