MOMENT OF A FORCE SCALAR FORMULATION, CROSS PRODUCT, MOMENT OF A FORCE VECTOR FORMULATION, & PRINCIPLE OF MOMENTS

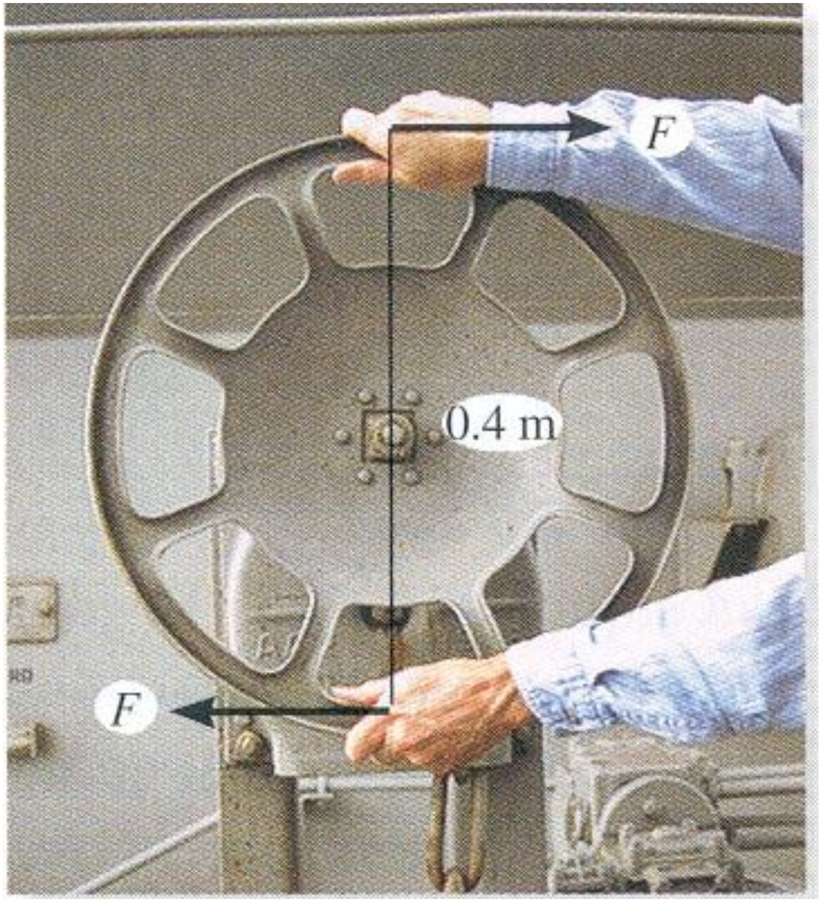
Today's Objectives :

Students will be able to:

a) understand and define moment, and,

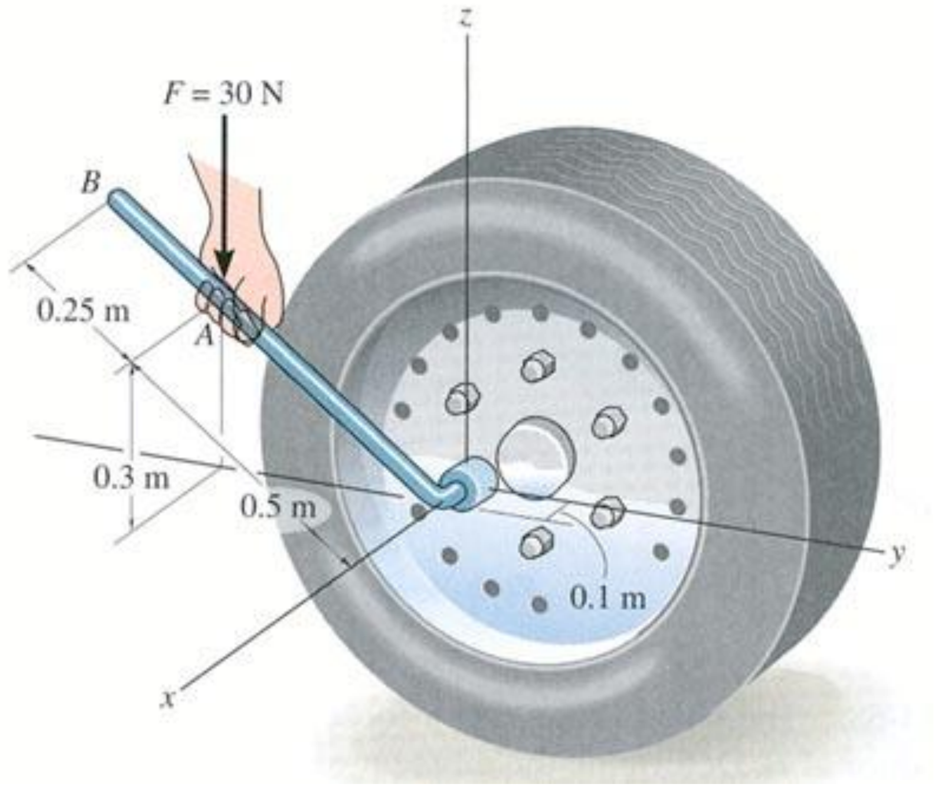
b) determine moments of a force in 2-D and 3-D cases.

APPLICATIONS



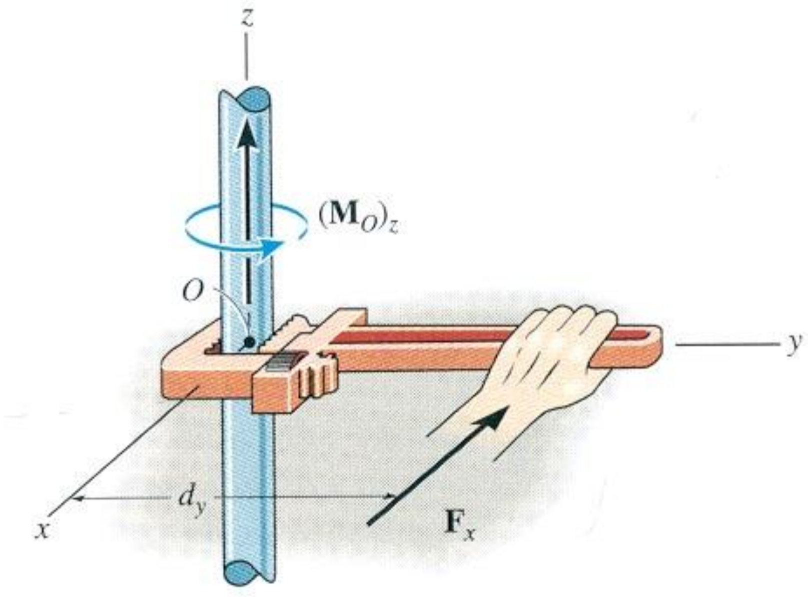
What is the net effect of the two forces on the wheel?

APPLICATIONS
(continued)



What is the effect of the 30 N force on the lug nut?

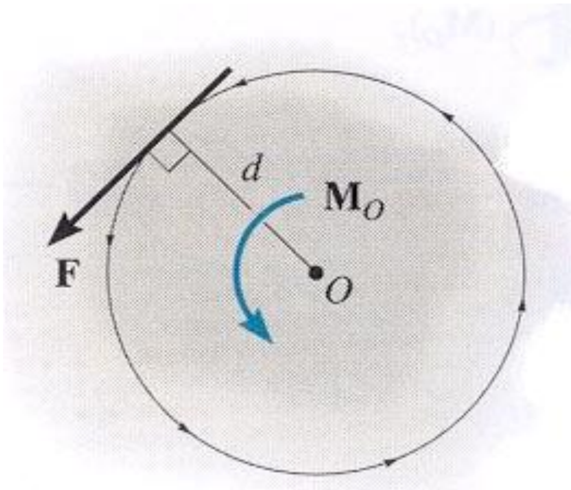
MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)

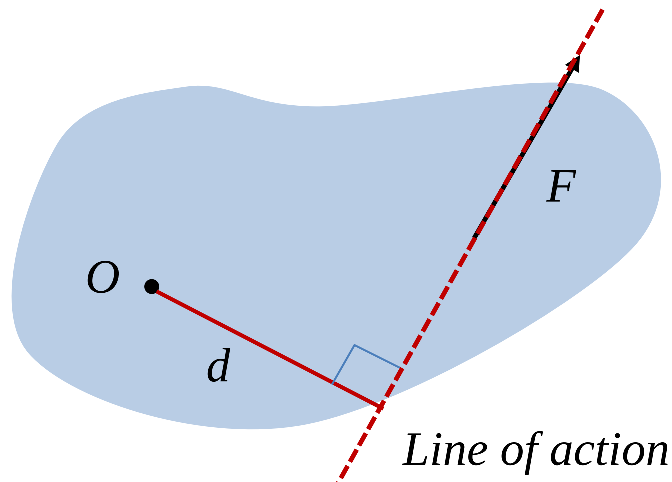


The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT OF A FORCE - SCALAR FORMULATION (continued)

In the 2-D case, the magnitude of the moment is $M_o = F\,d$

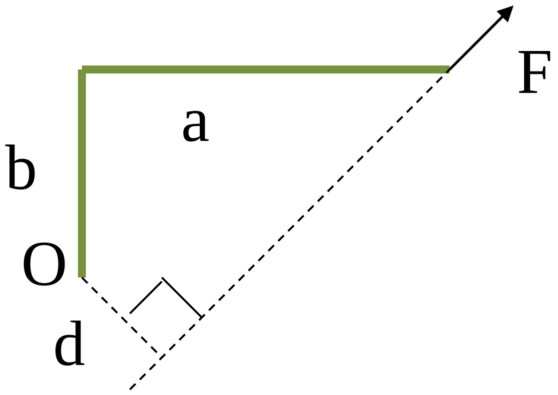




As shown, d is the perpendicular distance from point O to the line of action of the force.

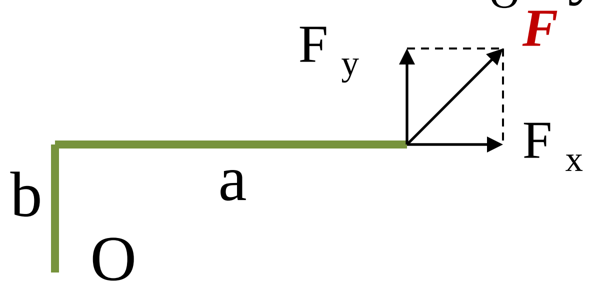
In 2-D, the direction of M_O is either clockwise or counter-clockwise depending on the tendency for rotation.

MOMENT OF A FORCE - SCALAR FORMULATION (continued)



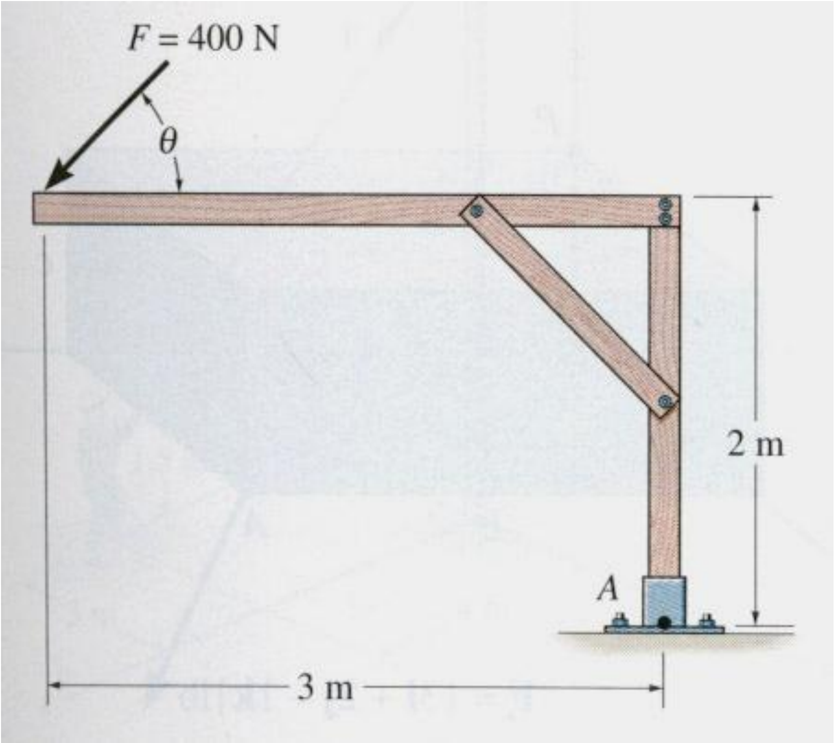
For example, $M_O = F\,d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of ***F*** as shown.



Using this approach, $M_O = (F_Y\, a) - (F_X\, b)$. Note the different signs on the terms! The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

EXAMPLE #1



Given: A 400 N force is applied to the frame and $\theta = 20°$.

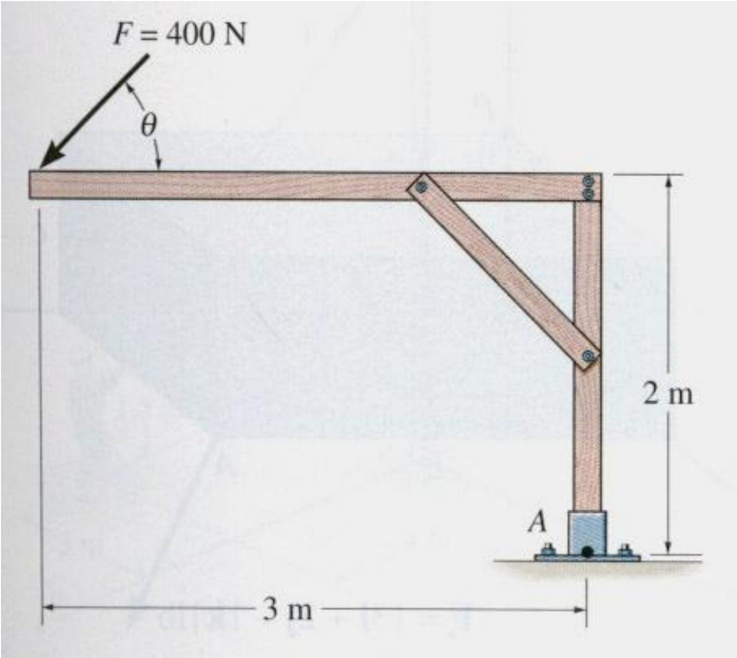
Find: The moment of the force at A.

Plan:

1) Resolve the force along x and y axes.

2) Determine M_A using scalar analysis.

EXAMPLE #1 (continued)



Solution

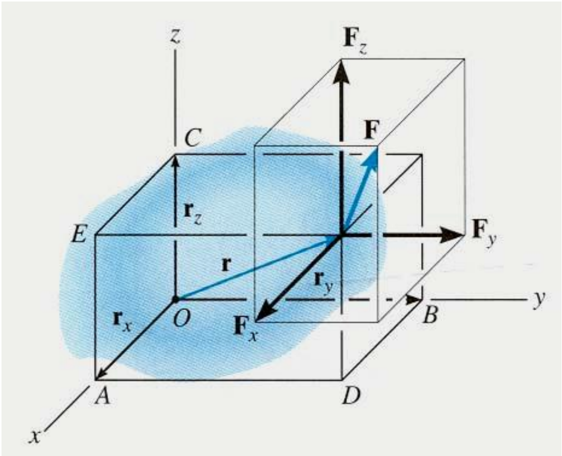
$+\uparrow F_y = -400 \cos 20° \text{ N}$

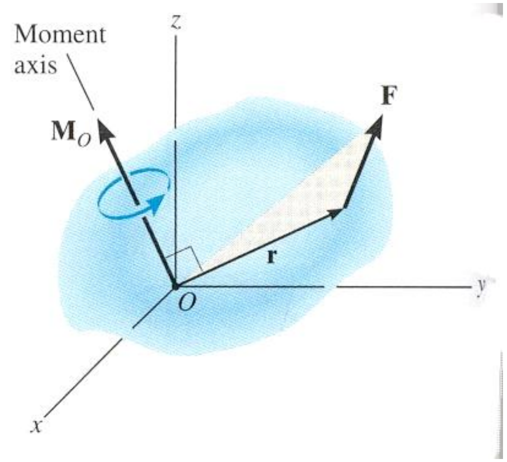
$+\rightarrow F_x = -400 \sin 20° \text{ N}$

$+\circlearrowleft M_A = \{(400 \cos 20°)(2) + (400 \sin 20°)(3)\} \text{ N·m}$

$= 1160 \text{ N·m}$

MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)



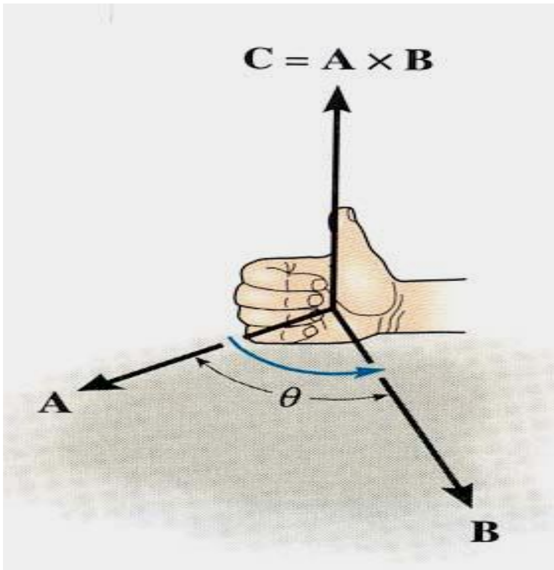


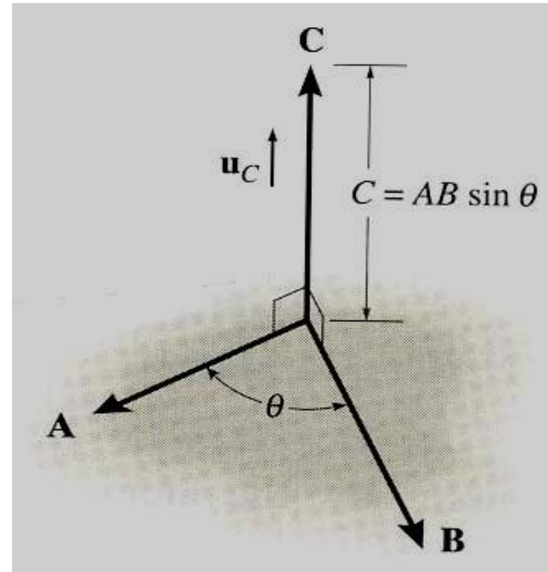
Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here $\boldsymbol{r}$ is the position vector from point O to any point on the line of action of $\boldsymbol{F}$. Need to review cross-product.

CROSS PRODUCT
(Section 4.2)





In general, the cross product of two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ results in another vector $\boldsymbol{C}$, i.e., $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \mathrm{A\,B} \sin\theta\, \boldsymbol{u_C}$$

Here $\boldsymbol{u_C}$ is the unit vector perpendicular to both A and B vectors as shown (or to the plane containing the A and B vectors).

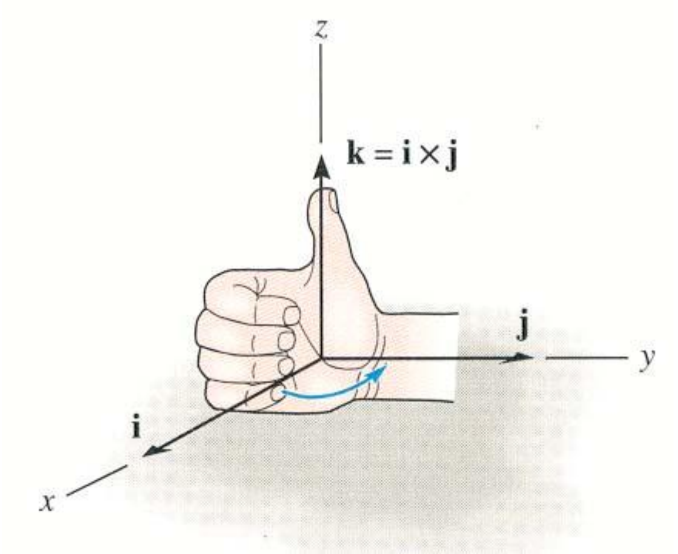
Note: $\vec{C} \perp \vec{A}$ & $\overrightarrow{C} \perp B$

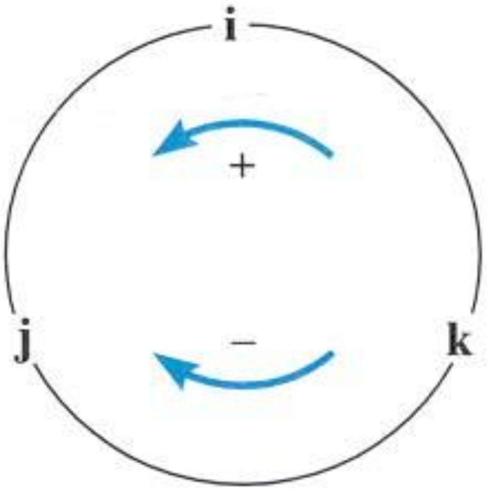
CROSS PRODUCT
(continued)

The right hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





CROSS PRODUCT
(continued)

You can evaluate the cross product of two vectors if you have them in Cartesian form.

$$
\begin{aligned}
\vec{C} &= \vec{A} \times \vec{B} \\
&= \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right) \\
&= A_xB_x\ \hat{\imath} \times \hat{\imath} + A_xB_y\ \hat{\imath} \times \hat{\jmath} + A_xB_z\ \hat{\imath} \times \hat{k} + \\
&\quad A_yB_x\ j \times \hat{\imath} + A_yB_y\ \hat{\jmath} \times \hat{\jmath} + A_yB_z\ \hat{\jmath} \times \hat{k} + \\
&\quad A_zB_x\ \hat{k} \times \hat{\imath} + A_zB_y\ \hat{k} \times \hat{\jmath} + A_zB_z\ \hat{k} \times \hat{k}
\end{aligned}
$$

But there is a simpler way to evaluate this.

CROSS PRODUCT
(continued)

Of even more utility, the cross product can be written as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

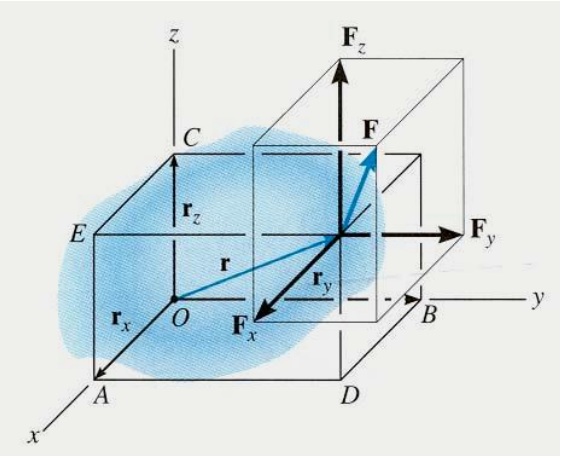
Each component can be determined using 2 × 2 determinants.

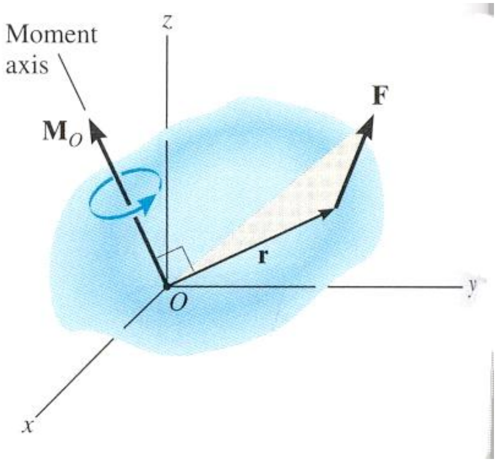
For element **i:** $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j:** $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

For element **k:** $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)





Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here ***r*** is the position vector from point O to any point on the line of action of ***F***.

MOMENT OF A FORCE – VECTOR FORMULATION
(continued)

So, using the cross product, a moment can be expressed as:

Always write this!

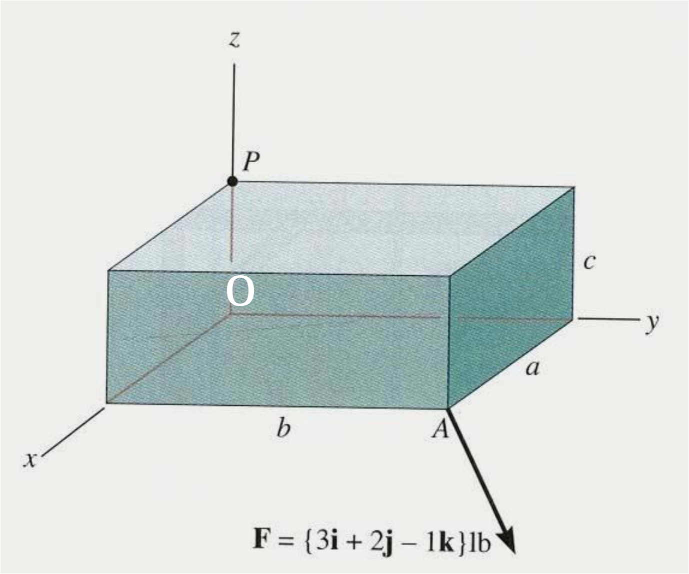
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y\ F_Z - r_Z\ F_y)\ \boldsymbol{i} - (r_x\ F_z - r_z\ F_x)\ \boldsymbol{j} + (r_x\ F_y - r_y\ F_x)\ \boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

EXAMPLE # 2



Given: a = 3 in, b = 6 in and c = 2 in.

Find: Moment of $\boldsymbol{F}$ about point O.

Plan:

1) Find $\boldsymbol{r}_{OA}$.

2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$.

Solution $\boldsymbol{r}_{OA} = \{3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 0\,\boldsymbol{k}\}$ in

$$\mathbf{M}_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\}\,\boldsymbol{i} - \{3(-1) - 0(3)\}\,\boldsymbol{j} + \{3(2) - 6(3)\}\,\boldsymbol{k}]\ \text{lb·in}$$

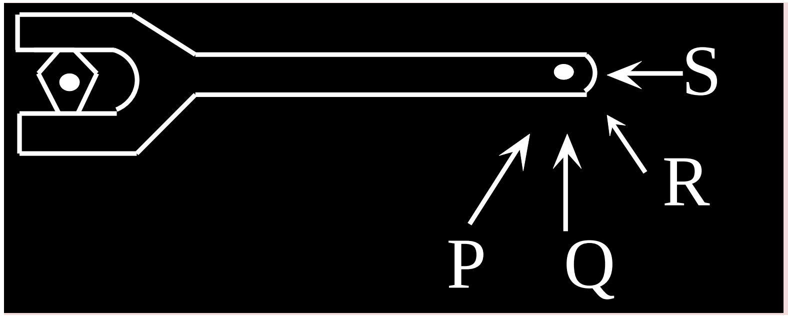
$$= \{-6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 12\,\boldsymbol{k}\}\ \text{lb·in}$$

CONCEPT QUIZ

1. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

A) (Q, P)   B) (R, S)

C) (P, R)   D) (Q, S)

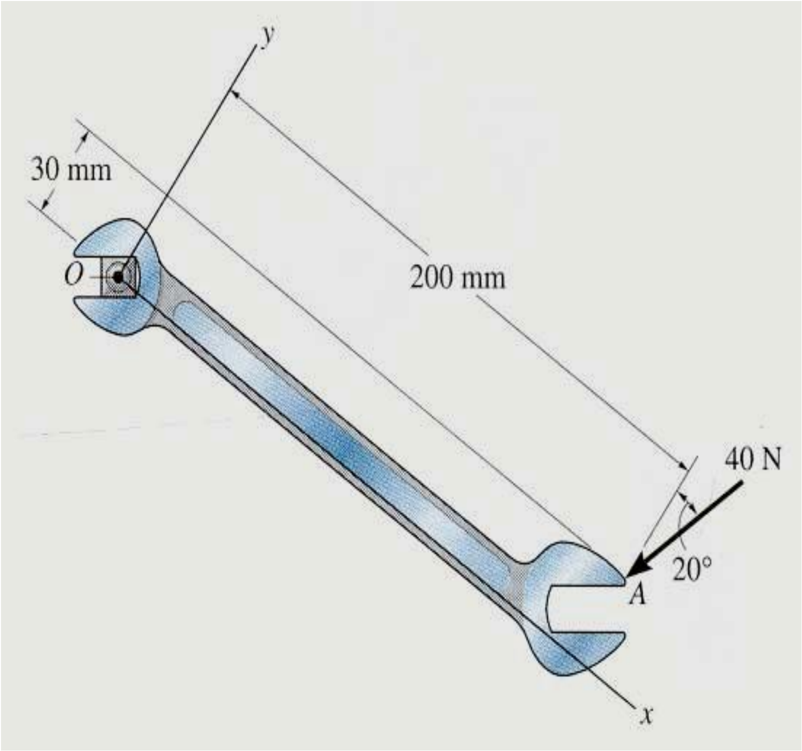


2. If $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$, then what will be the value of $\boldsymbol{M} \bullet \boldsymbol{r}$?

A) 0   B) 1

C) r^2 F   D) None of the above.

GROUP PROBLEM SOLVING



Given: A 40 N force is applied to the wrench.

Find: The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

2) Determine M_O using scalar analysis.

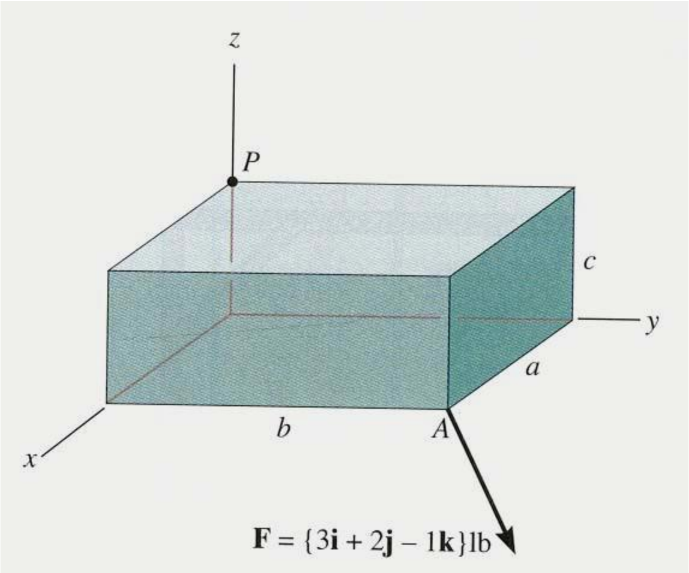
Solution: $+\uparrow F_y = -\,40 \cos 20° \text{ N}$

$+\rightarrow F_x = -\,40 \sin 20° \text{ N}$

$\circlearrowleft + \quad M_O = \{-(40 \cos 20°)(200) + (40 \sin 20°)(30)\} \text{N·mm}$

$= -7107 \text{ N·mm} = -\,7.11 \text{ N·m}$

GROUP PROBLEM SOLVING



Given: a = 3 in , b = 6 in and c = 2 in

Find: Moment of F about point P

Plan: 1) Find $\boldsymbol{r}_{PA}$.

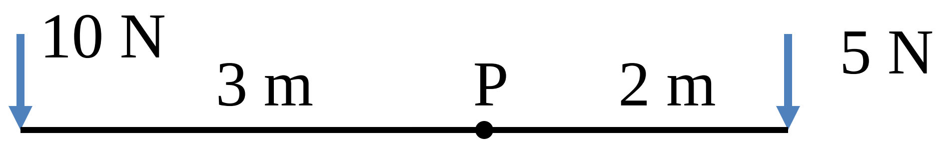
2) Determine $\boldsymbol{M}_P = \boldsymbol{r}_{PA} \times \boldsymbol{F}$

.

Solution: $\boldsymbol{r}_{PA} = \{ 3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 2\,\boldsymbol{k} \}$ in

$$\boldsymbol{M}_P = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & -2 \\ 3 & 2 & -1 \end{vmatrix} = \{ -2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 12\,\boldsymbol{k} \} \text{ lb} \cdot \text{in}$$

ATTENTION QUIZ



1. Using the CCW direction as positive, the net moment of the two forces about point P is

A) 10 N ·m B) 20 N ·m C) - 20 N ·m

D) 40 N ·m E) - 40 N ·m

2. If ***r*** = { 5 ***j*** } m and ***F*** = { 10 ***k*** } N, the moment

r* x *F equals { ________ } N·m.

A) 50 ***i*** B) 50 ***j*** C) –50 ***i***

D) – 50 ***j*** E) 0