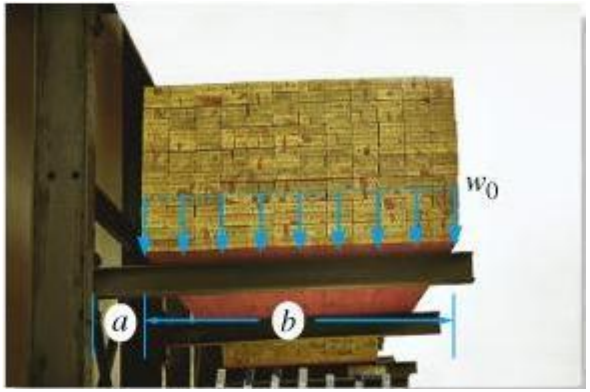
REDUCTION OF A SIMPLE DISTRIBUTED LOADING

Today's Objectives:

Students will be able to determine an equivalent force for a distributed load.



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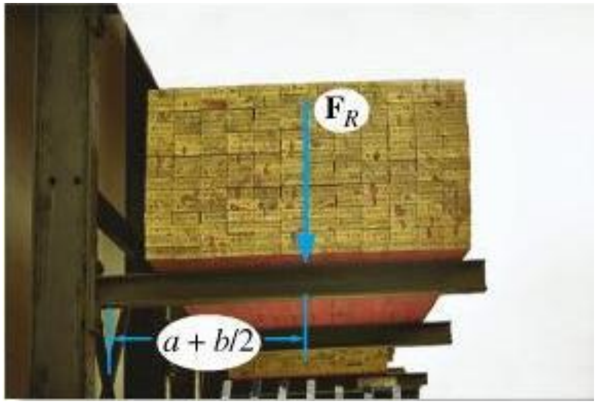
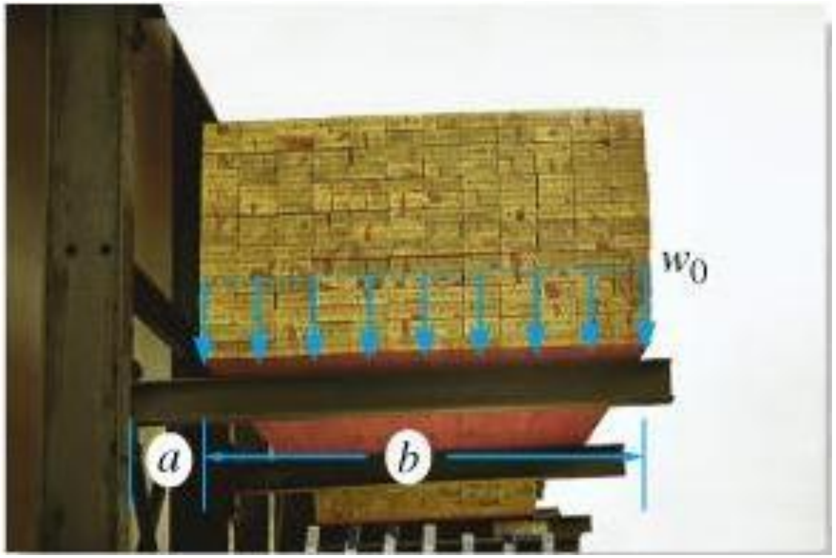


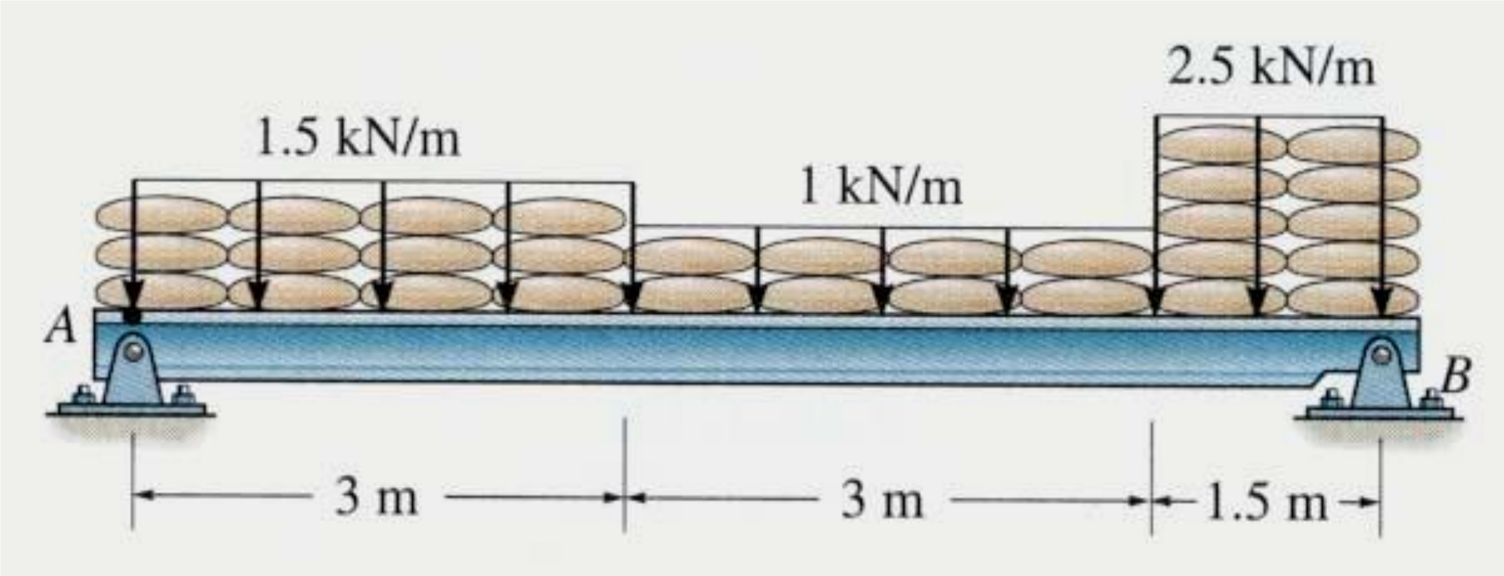
APPLICATIONS



A distributed load on the beam exists due to the weight of the lumber.

Is it possible to reduce this force system to a single force that will have the same external effect?
If yes, how?

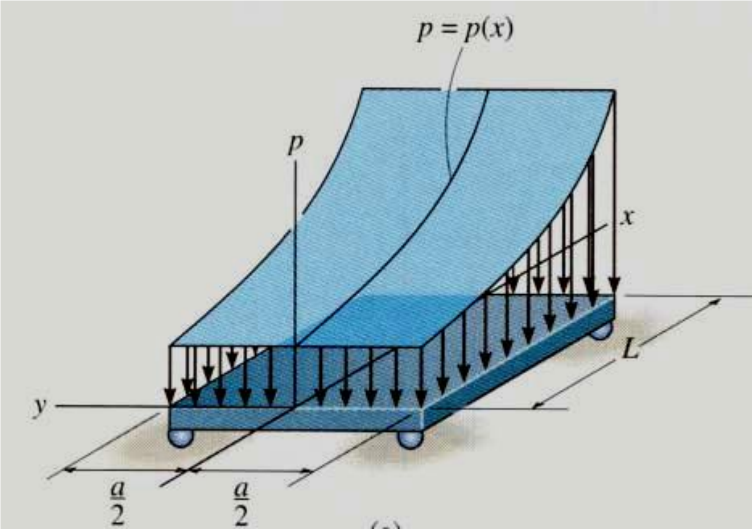
APPLICATIONS
(continued)

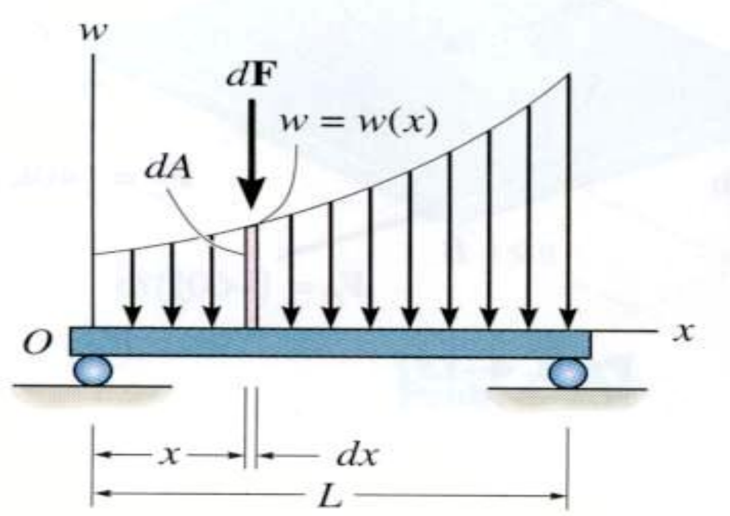


The sandbags on the beam create a distributed load.

How can we determine a single equivalent resultant force and its location?

DISTRIBUTED LOADING



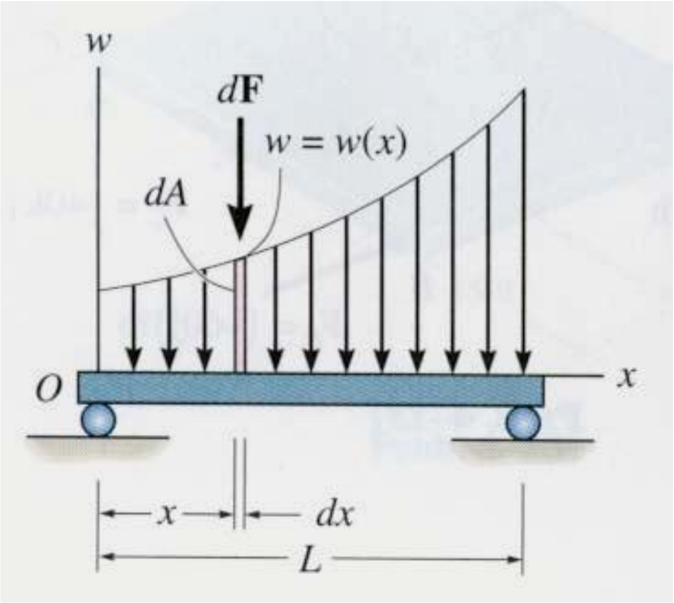


In many situations a surface area of a body is subjected to a distributed load. Such forces are caused by winds, fluids, or the weight of items on the body's surface.

We will analyze the most common case of a distributed pressure loading. This is a uniform load along one axis of a flat rectangular body.

In such cases, w is a function of x and has units of force per length.

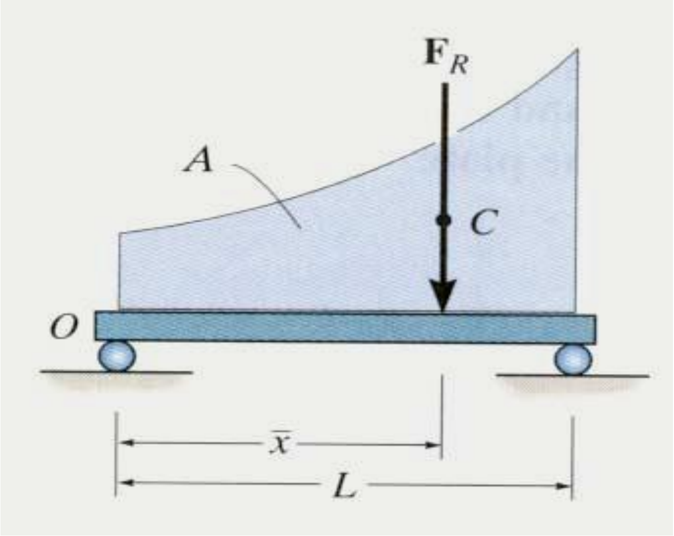
MAGNITUDE OF RESULTANT FORCE



Consider an element of length dx.

The force magnitude dF acting on it is given as

$$dF = w(x)\,dx$$

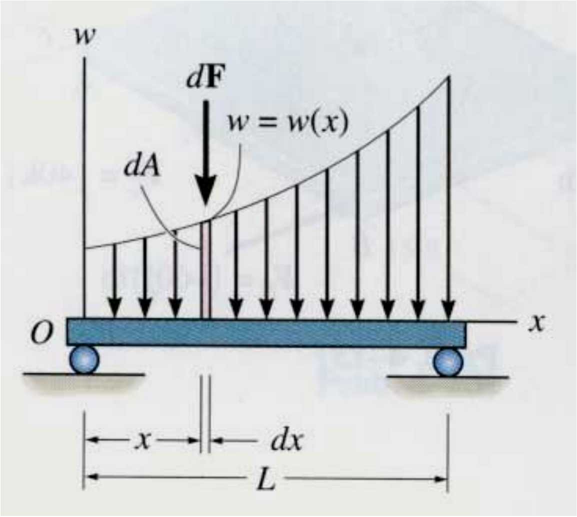


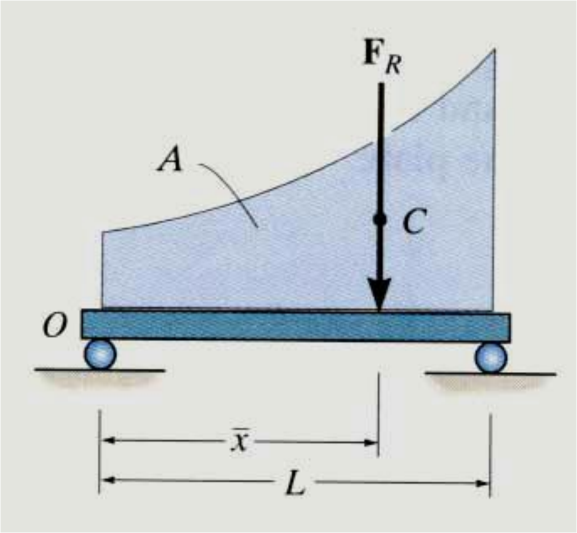
The net force on the beam is given by

$$+\downarrow\ F_R = \int_L dF = \int_L w(x)\,dx = A$$

Here A is the area under the loading curve w(x).

LOCATION OF THE RESULTANT FORCE





The force dF will produce a moment of (x)(dF) about point O.

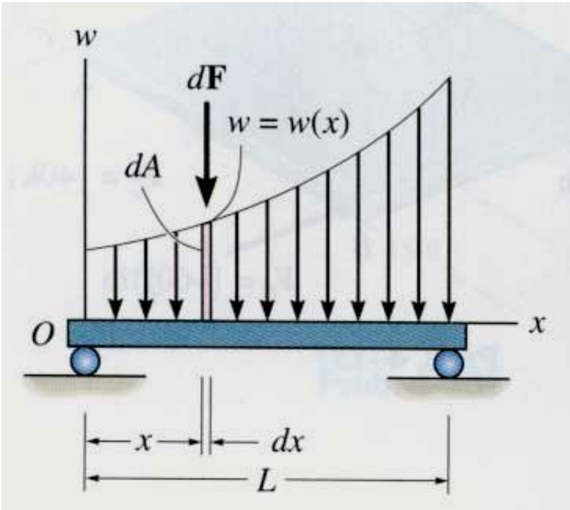
The total moment about point O is given as

$$\circlearrowleft + \; M_{RO} = \int_L x \, dF = \int_L x \, w(x) \, dx$$

Assuming that F_R acts at $\bar{x}$, it will produce the moment about point O as

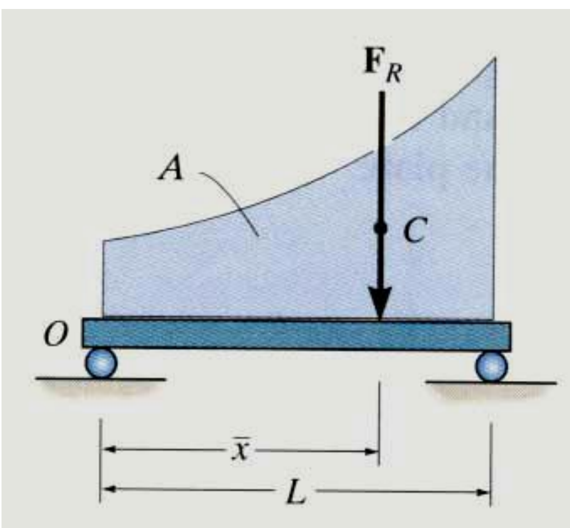
$$\circlearrowleft + \; M_{RO} = (\bar{x})\,(F_R) = \bar{x} \int_L w(x) \, dx$$

LOCATION OF THE RESULTANT FORCE (continued)



Comparing the last two equations, we get

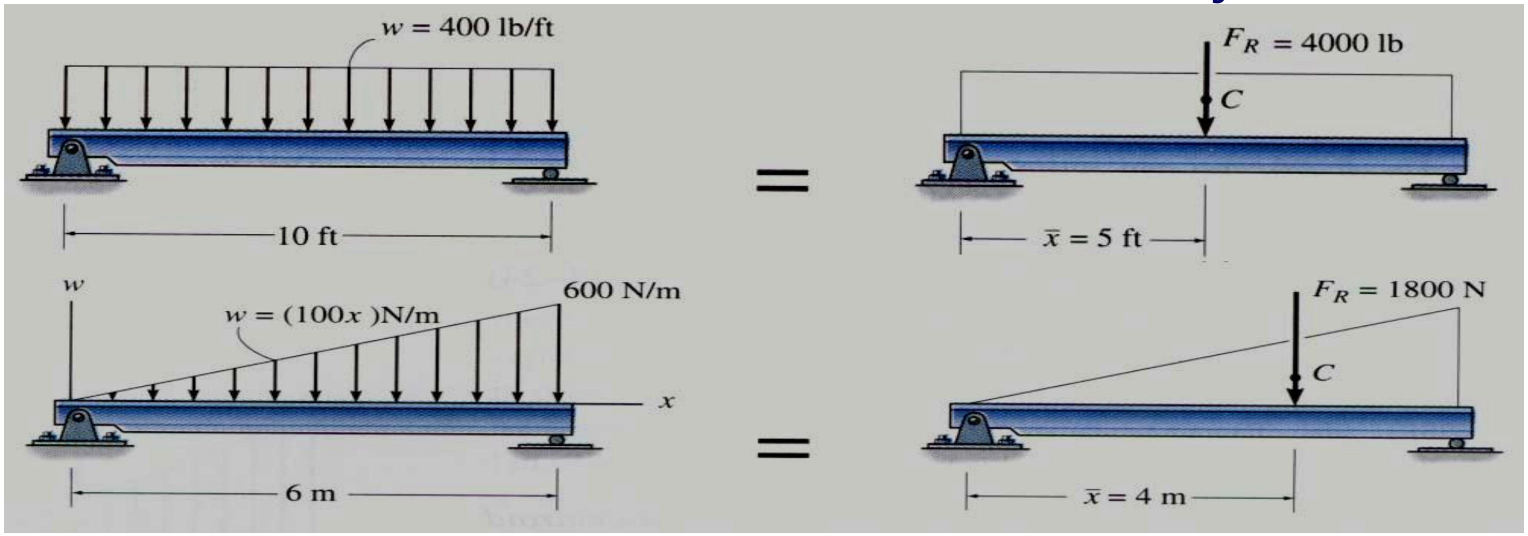
$$\bar{x} = \frac{\int_L x\, w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



You will learn later that F_R acts through a point "C," which is called the geometric center or centroid of the area under the loading curve w(x).

EXAMPLES

Until you learn more about centroids, we will consider only rectangular and triangular loading diagrams whose centroids are well defined and shown on the inside back cover of your textbook.

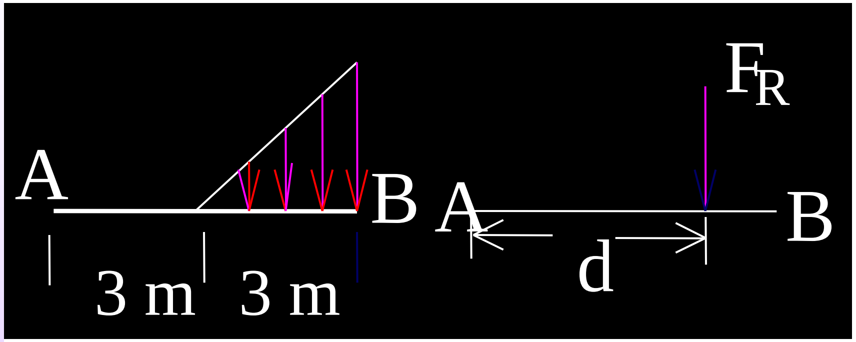


In a rectangular loading, $F_R = 400 \times 10 = 4{,}000$ lb and $\bar{x} = 5$ ft.

In a triangular loading , $F_R = (0.5)\,(6000)\,(6) = 1{,}800$ N and $\bar{x} = 6 - (1/3)\,6 = 4$ m.

Please note that the centroid in a right triangle is at a distance one third the width of the triangle as measured from its base.

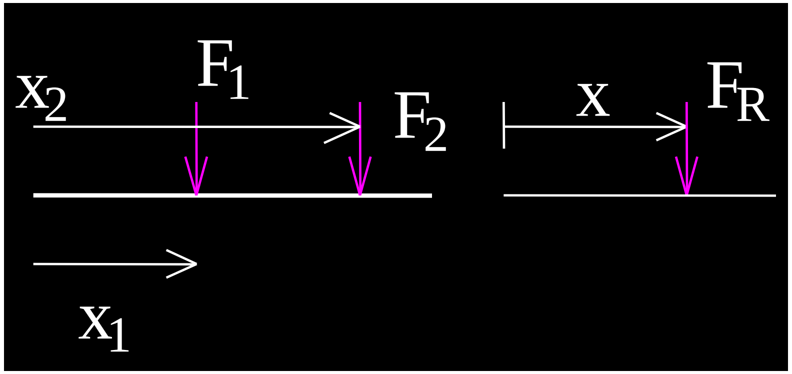
CONCEPT QUIZ



1. What is the location of F_R, i.e., the distance d?

A) 2 m B) 3 m C) 4 m

D) 5 m E) 6 m

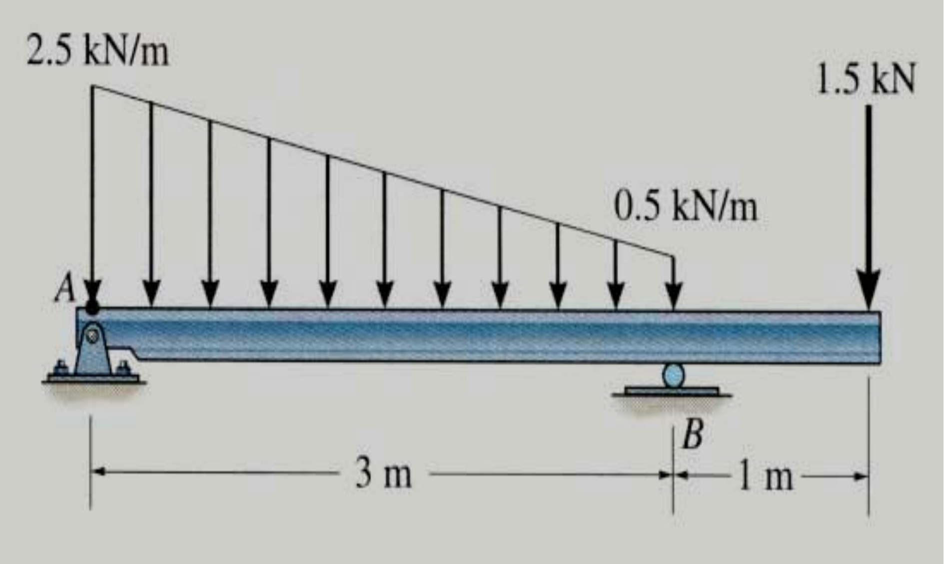


2. If $F_1 = 1$ N, $x_1 = 1$ m, $F_2 = 2$ N and $x_2 = 2$ m, what is the location of F_R, i.e., the distance x?

A) 1 m B) 1.33 m C) 1.5 m

D) 1.67 m E) 2 m

GROUP PROBLEM SOLVING



Given: The loading on the beam as shown.

Find: The equivalent force and its location from point A.

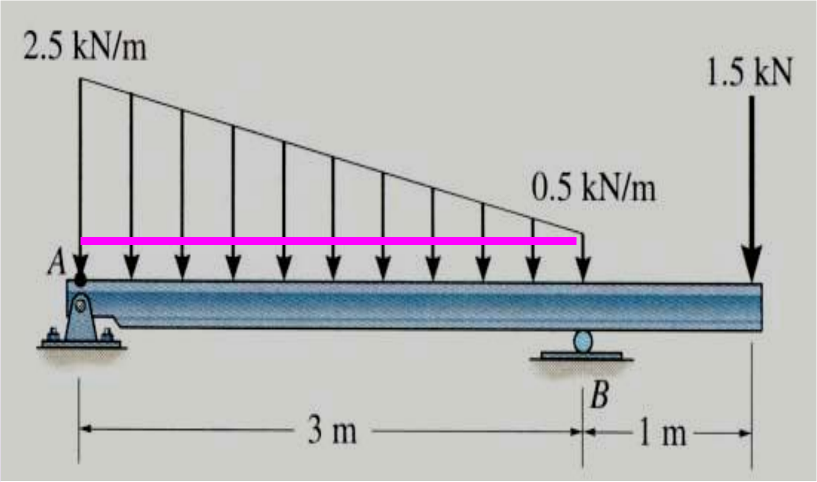
Plan:

1) Consider the trapezoidal loading as two separate loads (one rectangular and one triangular).

2) Find F_R and $\bar{x}$ for each of the two distributed loads.

3) Determine the overall F_R and for the three point loadings.

GROUP PROBLEM SOLVING (continued)



For the rectangular loading of height 0.5 kN/m and width 3 m,

$F_{R1} = 0.5 \text{ kN/m} \times 3 \text{ m} = 1.5 \text{ kN}$

$= 1.5 \text{ m from A}$

For the triangular loading of height 2 kN/m and width 3 m,

$F_{R2} = (0.5)(2 \text{ kN/m})(3 \text{ m}) = 3 \text{ kN}$

and its line of action is at $\bar{x} = 1 \text{ m}$ from A

For the combined loading of the three forces,

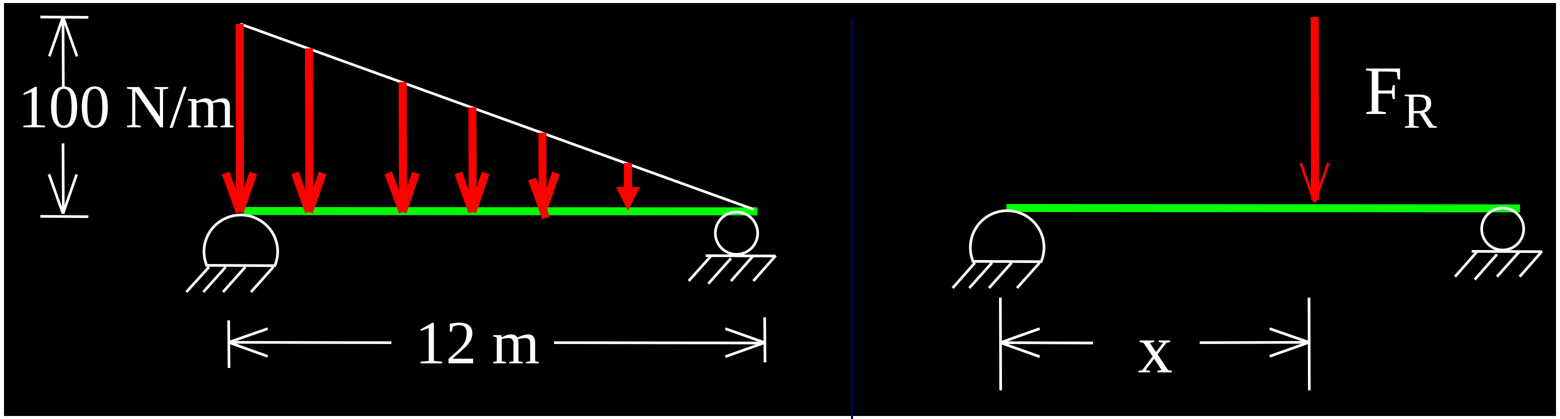
$F_R = 1.5 \text{ kN} + 3 \text{ kN} + 1.5 \text{ kN} = 6 \text{ kN}$

$\curvearrowleft + M_{RA} = (1.5)(1.5) + 3(1) + (1.5)\,4 = 11.25 \text{ kN} \bullet \text{m}$

Now, $F_R\,\bar{x} = 11.25 \text{ kN} \bullet \text{m}$

Hence, $\bar{x} = (11.25) / (6) = 1.88 \text{ m}$ from A.

ATTENTION QUIZ



1. F_R = ______________

A) 12 N B) 100 N

C) 600 N D) 1200 N

2. x = ______________.

A) 3 m B) 4 m

C) 6 m D) 8 m