EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM & COPLANAR FORCE SYSTEMS

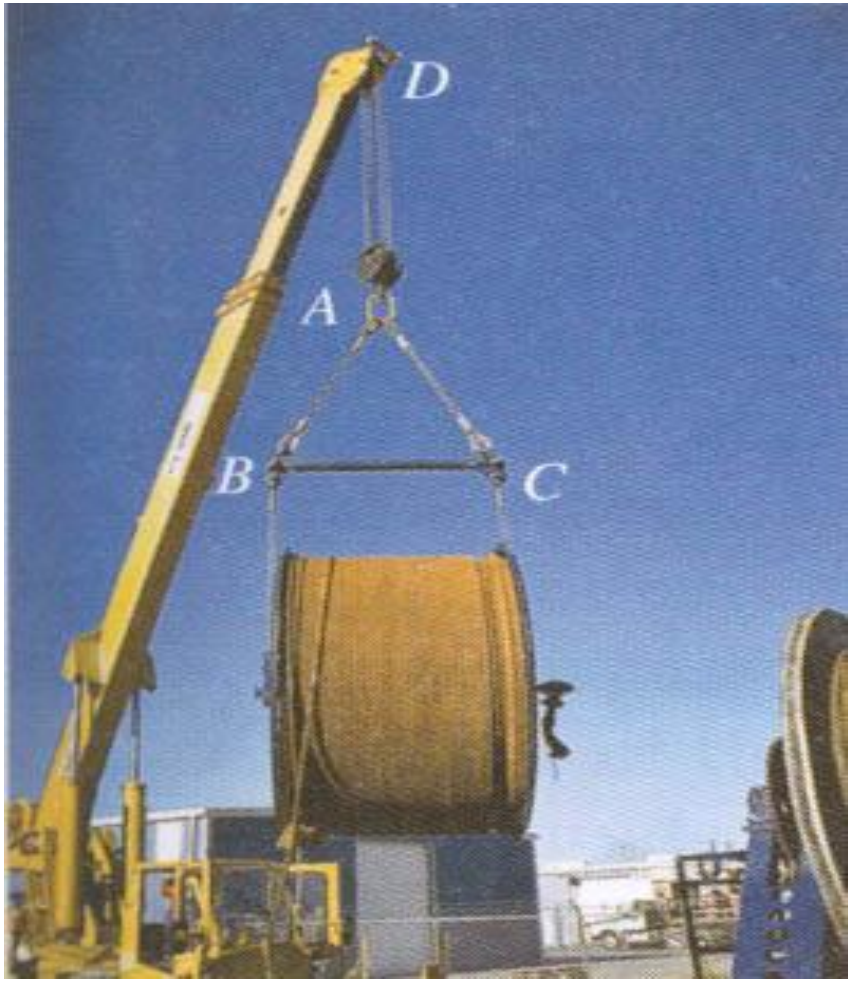
Today's Objectives:

Students will be able to :

a) Draw a free body diagram (FBD), and,

b) Apply equations of equilibrium to solve a 2-D problem.

APPLICATIONS

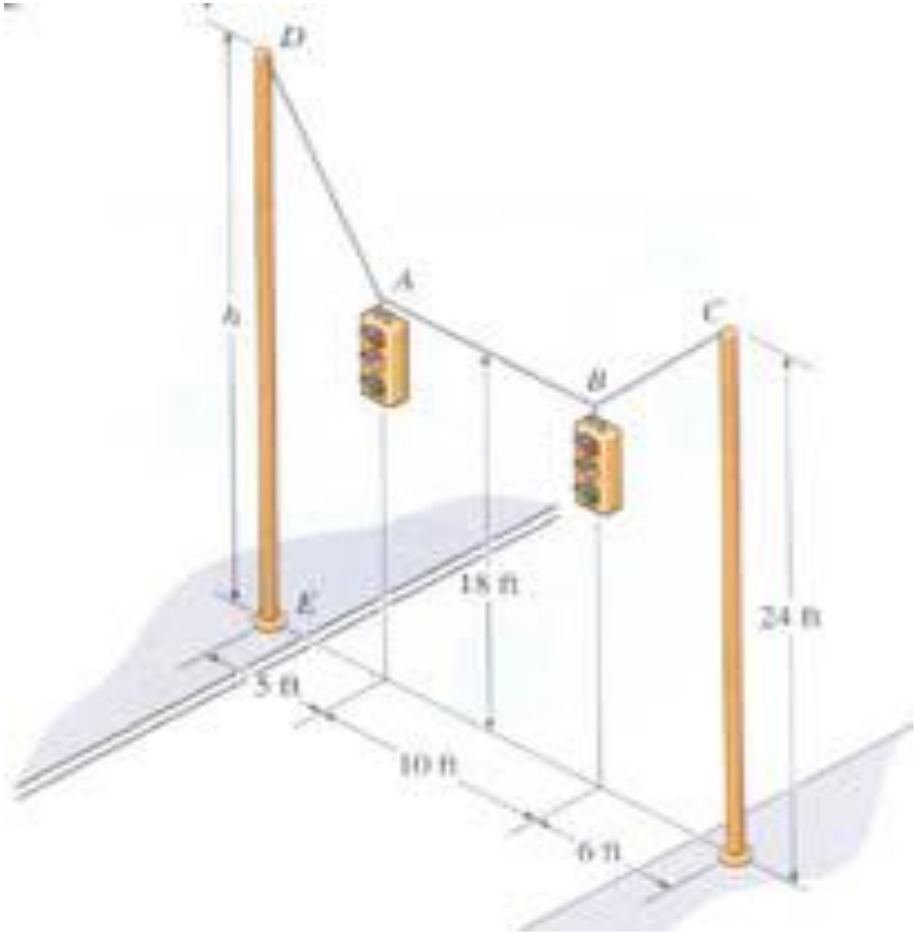


For a spool of given weight, what are the forces in cables AB and AC ?

APPLICATIONS
(continued)

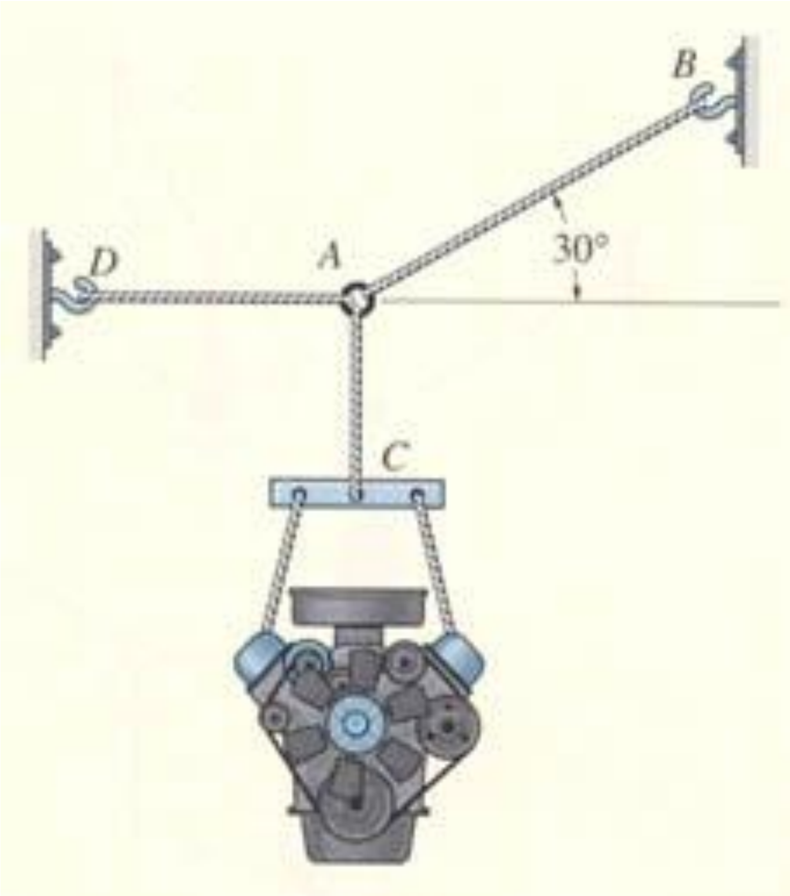
For a given cable strength, what is the maximum weight that can be lifted ?

APPLICATIONS
(continued)



For a given weight of the lights, what are the forces in the cables? What size of cable must you use ?

COPLANAR FORCE SYSTEMS
(Section 3.3)



This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium.

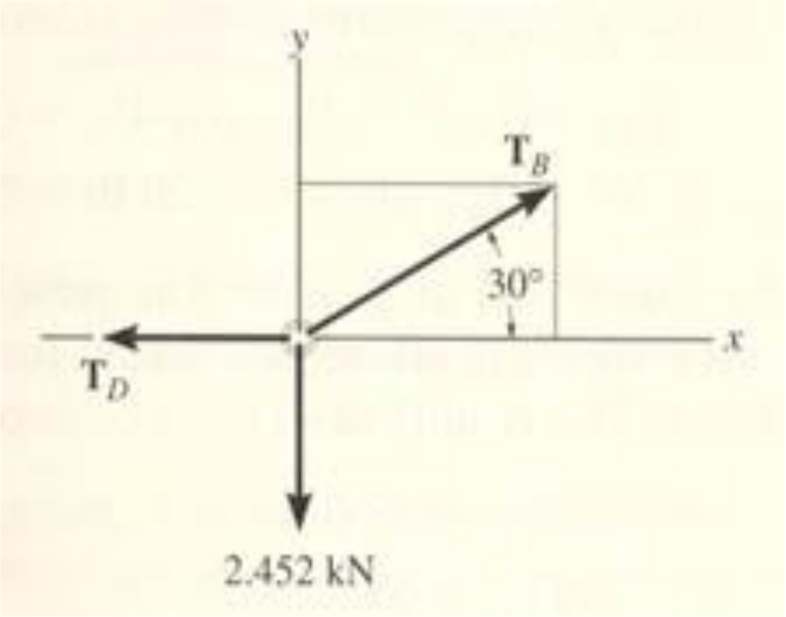
To determine the tensions in the cables for a given weight of the engine, we need to learn how to draw a free body diagram and apply equations of equilibrium.

THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

Free Body Diagrams are one of the most important things for you to know how to draw and use.

What ? - It is a drawing that shows all external forces acting on the particle.

Why ? - It helps you write the equations of equilibrium used to solve for the unknowns (usually forces or angles).



How ?

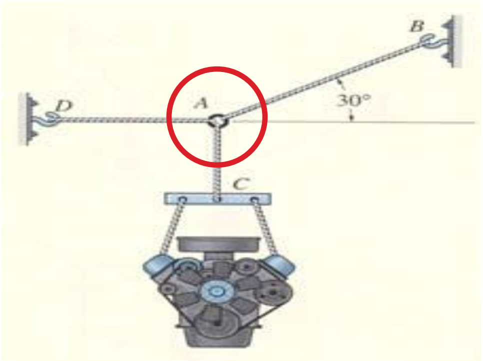
1. Imagine the particle to be isolated or cut free from its surroundings.

2. Show all the forces that act on the particle.

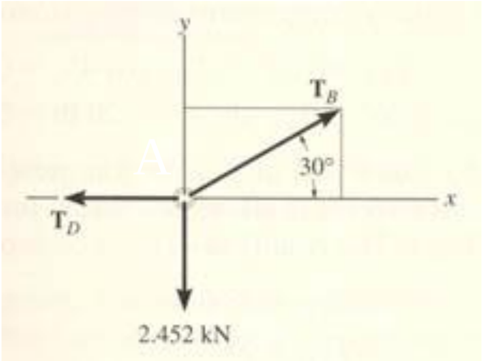
   Active forces: They want to move the particle.

   Reactive forces: They tend to resist the motion.

3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables .

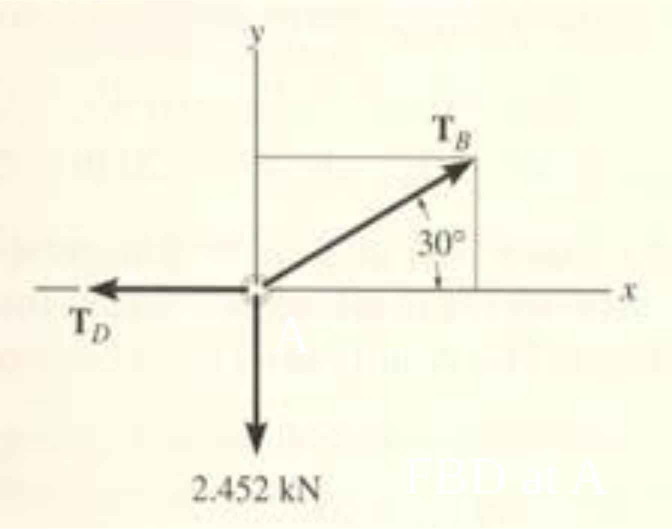


Note : Engine mass = 250 Kg



FBD at A

EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F_{AB}} + \boldsymbol{F_{AC}} + \boldsymbol{F_{AD}} = 0$

or $\Sigma \boldsymbol{F} = 0$

In general, for a particle in equilibrium, $\Sigma \boldsymbol{F} = 0$ or

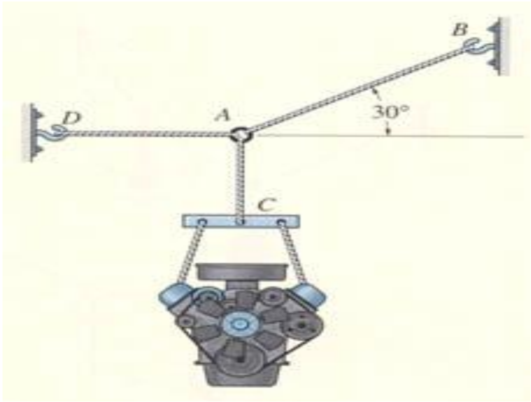
$\Sigma F_x \, \boldsymbol{i} + \Sigma F_y \, \boldsymbol{j} = 0 = 0 \, \boldsymbol{i} + 0 \, \boldsymbol{j}$ (A vector equation)

Or, written in a scalar form,

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

These are two scalar equations of equilibrium (EofE). They can be used to solve for up to two unknowns.

EXAMPLE



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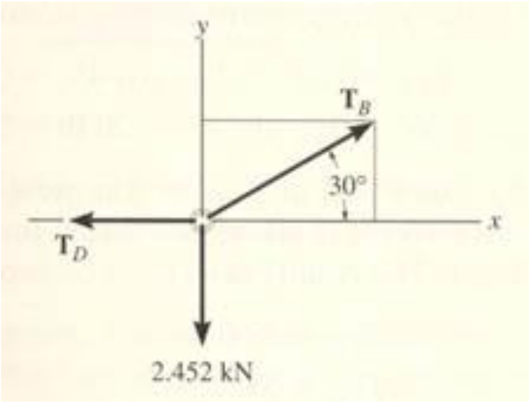


FBD at A

Write the scalar EofE:

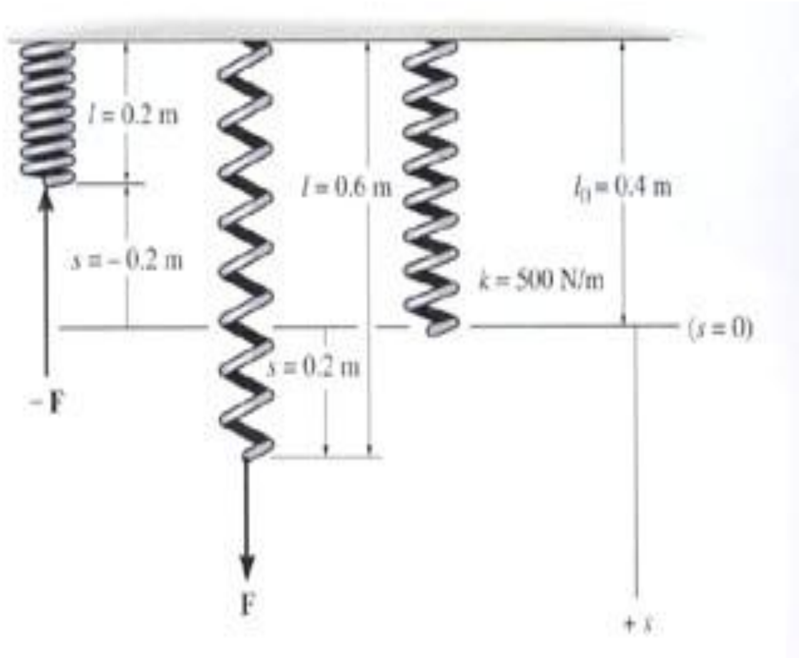
$$+ \rightarrow \Sigma F_x = T_B \cos 30^o - T_D = 0$$

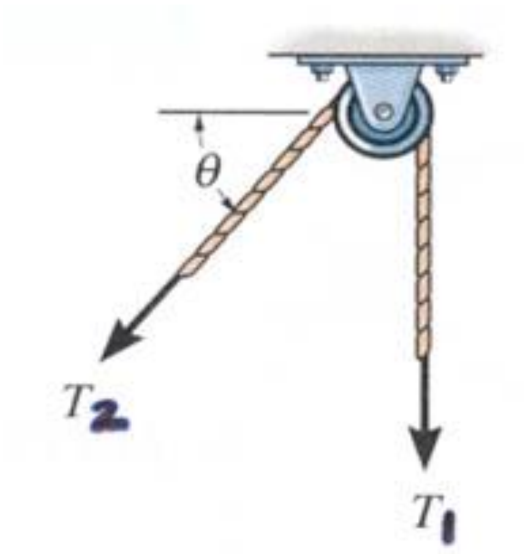
$$+ \uparrow \Sigma F_y = T_B \sin 30^o - 2.452 \text{ kN} = 0$$

Solving the second equation gives: $T_B = 4.90$ kN

From the first equation, we get: $T_D = 4.25$ kN

SPRINGS, CABLES, AND PULLEYS





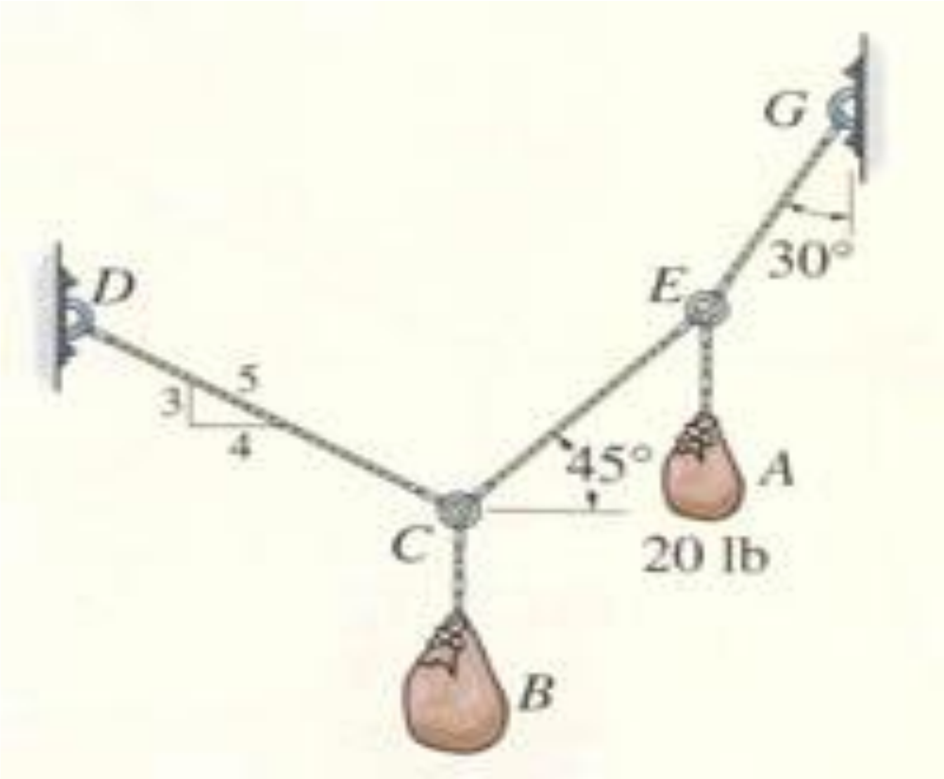
Cable is in tension

Spring Force = spring constant * deformation, or

$$F = k * S$$

With a frictionless pulley, $T_1 = T_2$.

EXAMPLE



Given: Sack A weighs 20 lb. and geometry is as shown.

Find: Forces in the cables and weight of sack B.

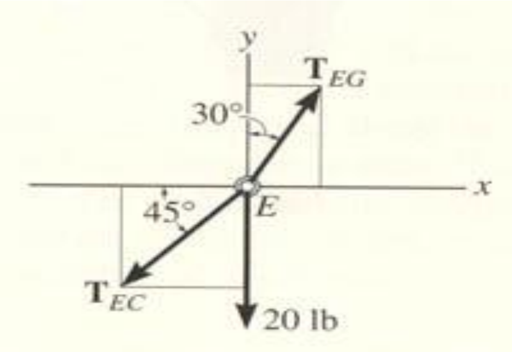
Plan:

1. Draw a FBD for Point E.

2. Apply EofE at Point E to solve for the unknowns (T_{EG} & T_{EC}).

3. Repeat this process at C.

EXAMPLE
(continued)



A FBD at E should look like the one to the left. Note the assumed directions for the two cable tensions.

The scalar E-of-E are:

$$+\rightarrow \quad \Sigma F_x = T_{EG} \sin 30^o - T_{EC} \cos 45^o = 0$$

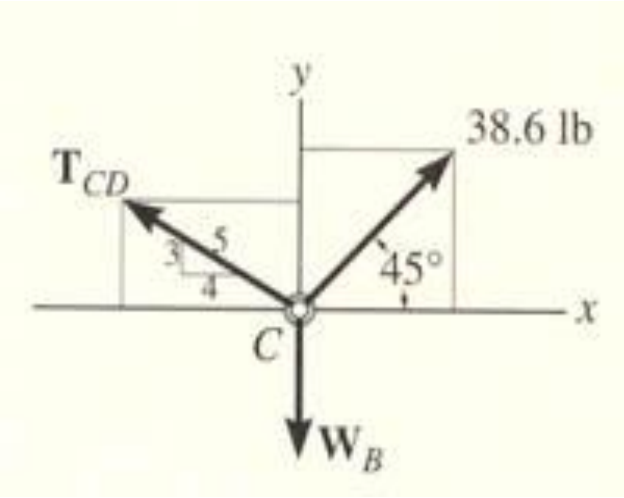
$$+\uparrow \quad \Sigma F_y = T_{EG} \cos 30^o - T_{EC} \sin 45^o - 20 \text{ lbs} = 0$$

Solving these two simultaneous equations for the two unknowns yields:

$T_{EC} = 38.6$ lb

$T_{EG} = 54.6$ lb

EXAMPLE (continued)



Now move on to ring C. A FBD for C should look like the one to the left.

The scalar E-of-E are:

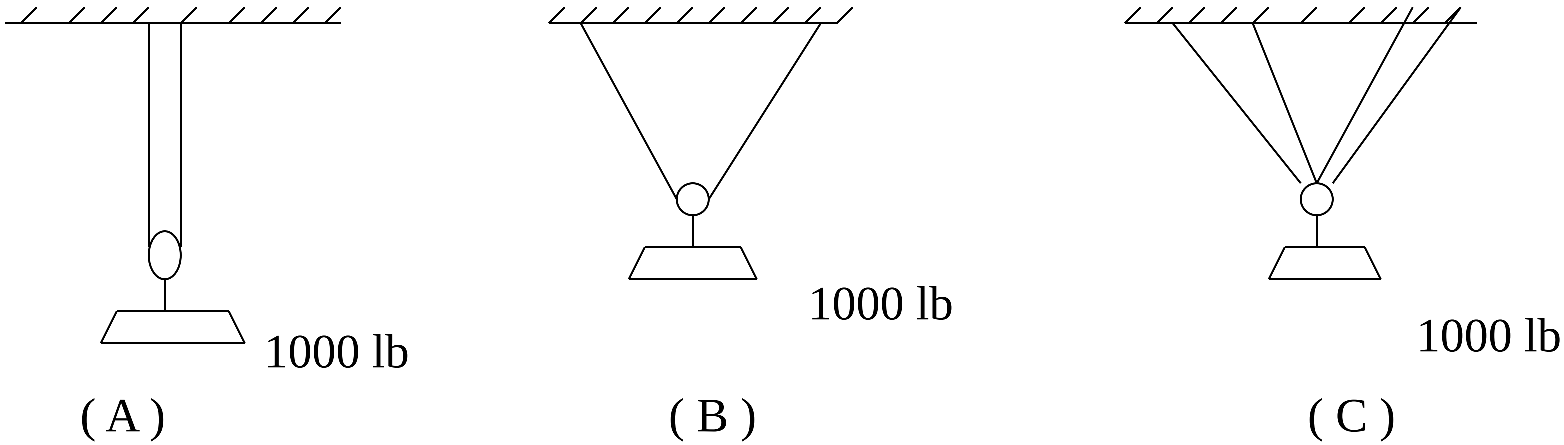
$$+\rightarrow \Sigma F_x = 38.64 \cos 45° - (4/5)\, T_{CD} = 0$$

$$+\uparrow \Sigma F_y = (3/5)\, T_{CD} + 38.64 \sin 45° - W_B = 0$$

Solving the first equation and then the second yields

$T_{CD} = 34.2$ lb and $W_B = 47.8$ lb .

CONCEPT QUESTIONS



1) Assuming you know the geometry of the ropes, you cannot determine the forces in the cables in which system above?

2) Why?

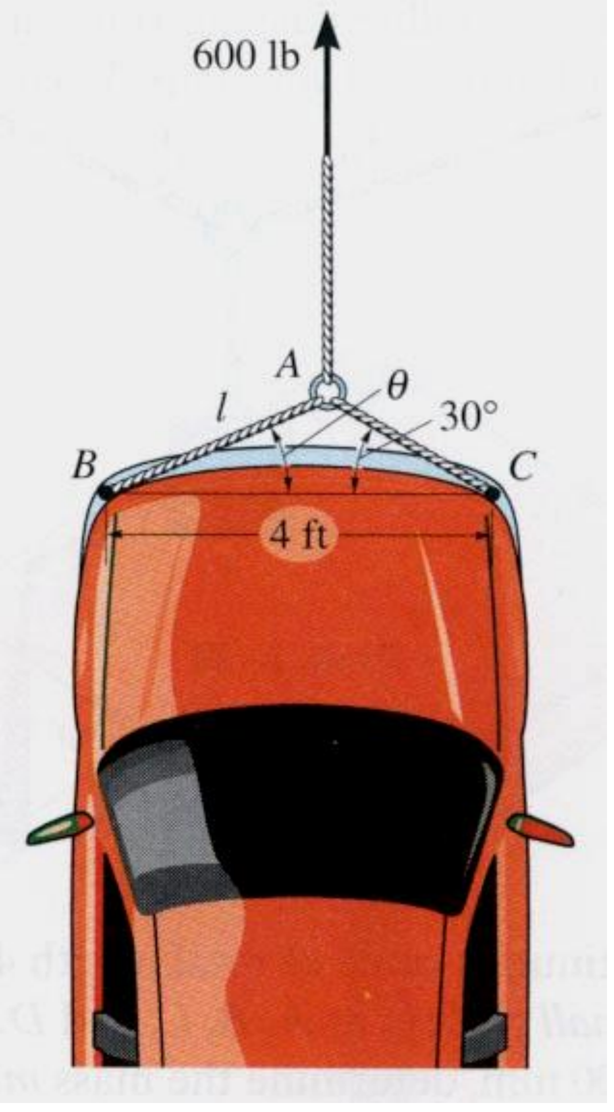
A) The weight is too heavy.

B) The cables are too thin.

C) There are more unknowns than equations.

D) There are too few cables for a 1000 lb weight.

GROUP PROBLEM SOLVING



Given: The car is towed at constant speed by the 600 lb force and the angle θ is 25 .

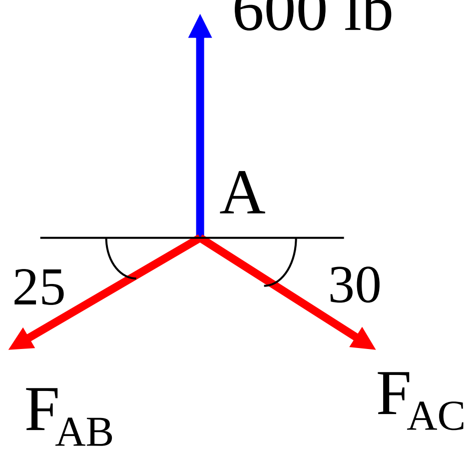
Find: The forces in the ropes AB and AC.

Plan:

1. Draw a FBD for point A.

2. Apply the E-of-E to solve for the forces in ropes AB and AC.

GROUP PROBLEM SOLVING
(continued)



FBD at point A

Applying the scalar E-of-E at A, we get;

$+ \rightarrow \sum F_x = F_{AC} \cos 30 \; - F_{AB} \cos 25 \; = 0$

$+ \rightarrow \sum F_y = -F_{AC} \sin 30 \; - F_{AB} \sin 25 \; + 600 \; = 0$

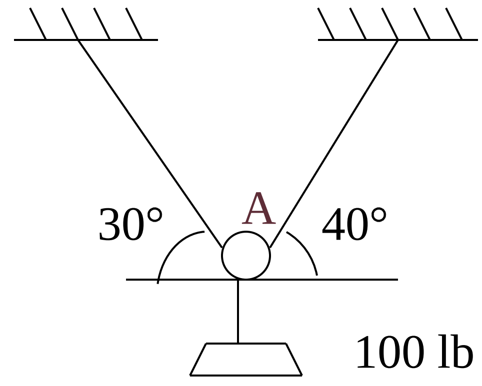
Solving the above equations, we get;

$F_{AB} = 634$ lb

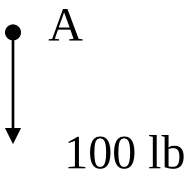
$F_{AC} = 664$ lb

ATTENTION QUIZ

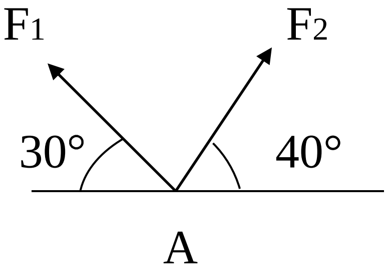
1. Select the correct FBD of particle A.



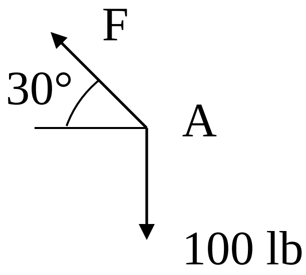
A)



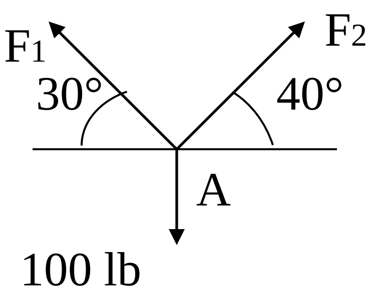
B)



C)

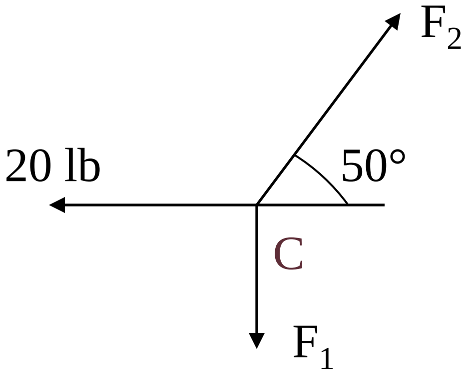


D)



ATTENTION QUIZ

2. Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is ___ . Use a sign convention of + $\rightarrow$.



A) $F_2 \sin 50° - 20 = 0$

B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$