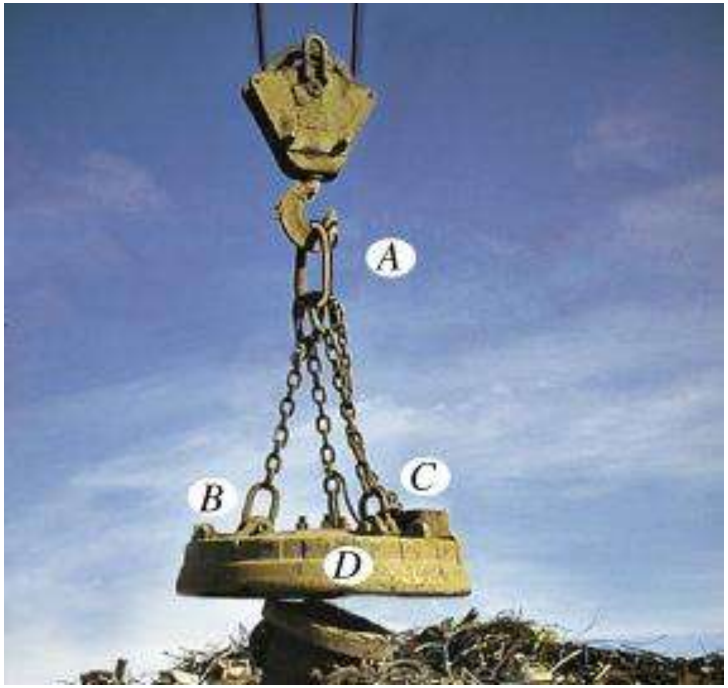
THREE-DIMENSIONAL FORCE SYSTEMS

Today's Objectives:

Students will be able to solve 3-D particle equilibrium problems by

a) Drawing a 3-D free body diagram, and,

b) Applying the three scalar equations (based on one vector equation) of equilibrium.



QUIZ

1. Particle P is in equilibrium with five (5) forces acting on it in 3-D space. How many scalar equations of equilibrium can be written for point P?

   A) 2　　B) 3　　C) 4

   D) 5　　E) 6

2. In 3-D, when a particle is in equilibrium, which of the following equations apply?

   A) $(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$

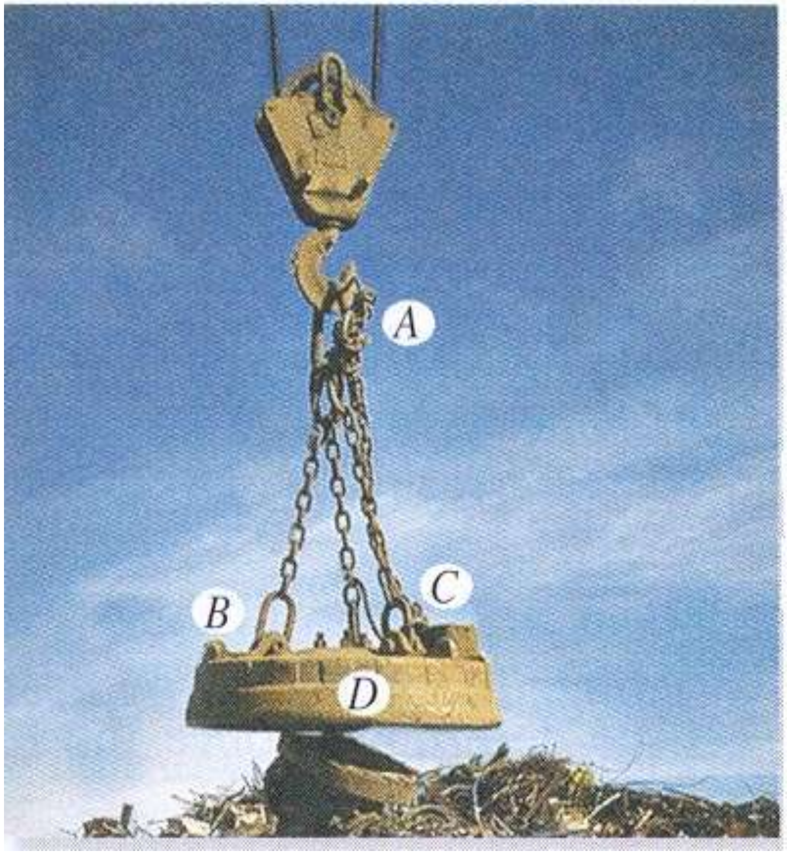
   B) $\Sigma \boldsymbol{F} = 0$

   C) $\Sigma F_x = \Sigma F_y = \Sigma F_z = 0$

   D) All of the above.

   E) None of the above.

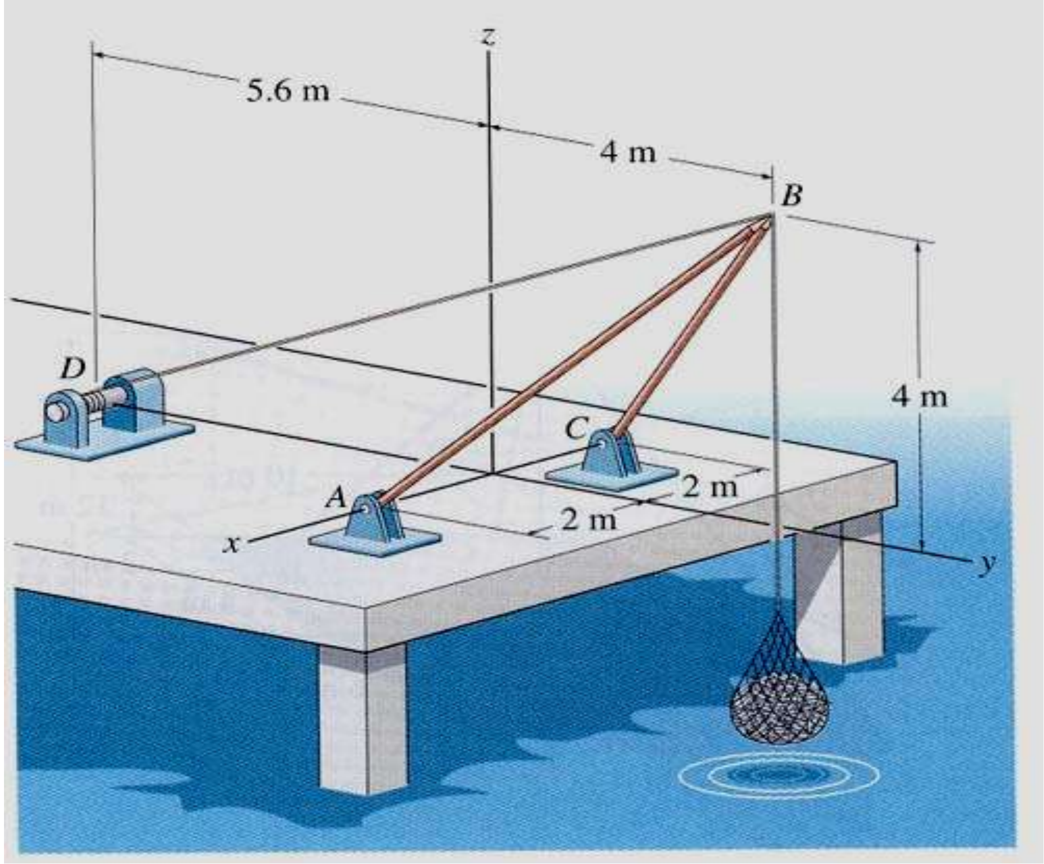
APPLICATIONS



The weights of the electromagnet and the loads are given.

Can you determine the forces in the chains?

APPLICATIONS
(continued)

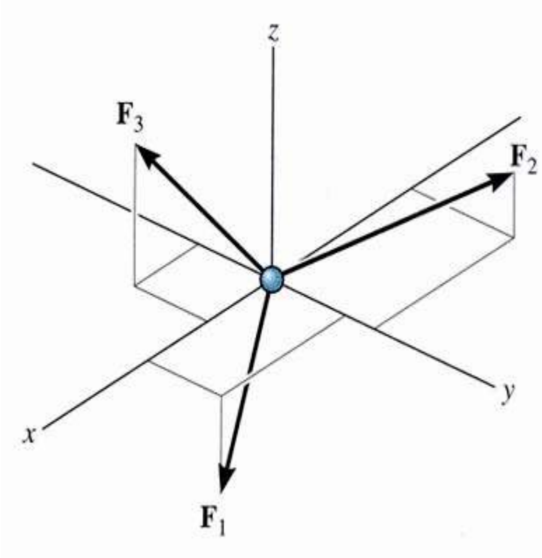


The shear leg derrick is to be designed to lift a maximum of 500 kg of fish.

What is the effect of different offset distances on the forces in the cable and derrick legs?

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \boldsymbol{F} = 0$).



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$$

This vector equation will be satisfied only when

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

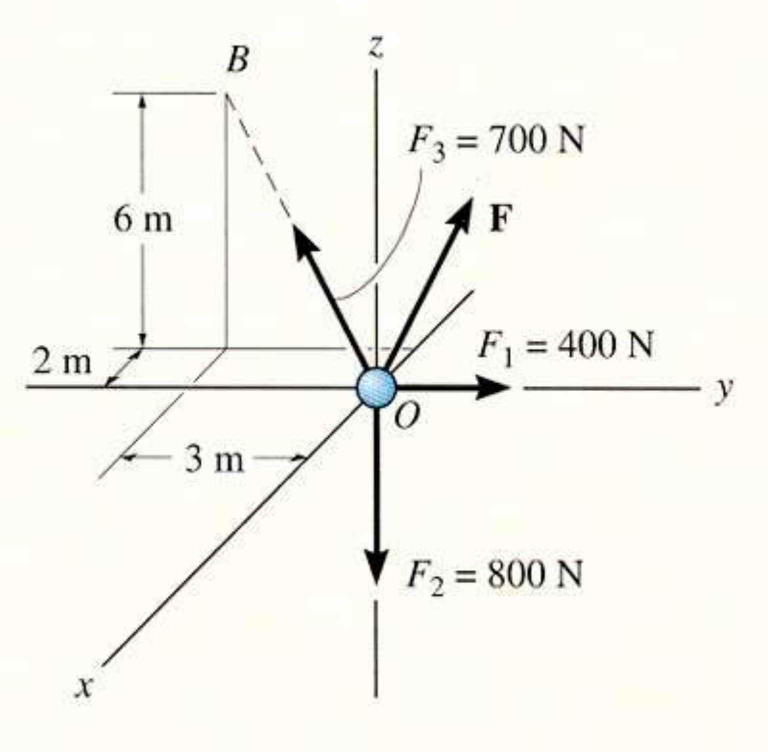
$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid at any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE #1

Given: $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$.

Find: The force $\boldsymbol{F}$ required to keep particle O in equilibrium.



<u>Plan</u>:

1) Draw a FBD of particle O.

2) Write the unknown force as

$$\boldsymbol{F} = \{F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k}\}\text{ N}$$

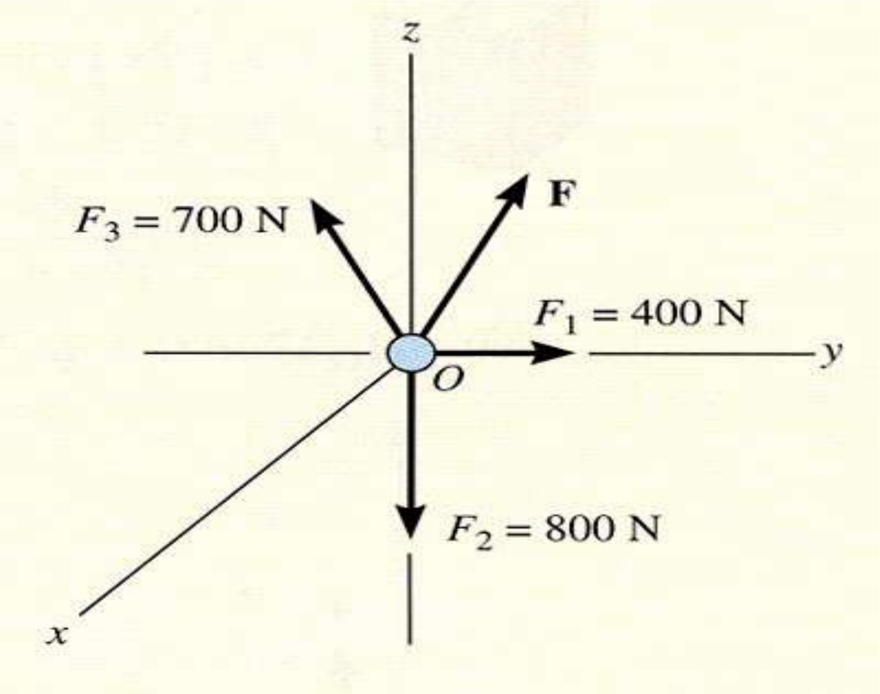
3) Write $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ in Cartesian vector form.

4) Apply the three equilibrium equations to solve for the three unknowns F_x, F_y, and F_z.

EXAMPLE #1
(continued)

$\boldsymbol{F_1} = \{400\ \boldsymbol{j}\}$N

$\boldsymbol{F_2} = \{-800\ \boldsymbol{k}\}$N



$$\boldsymbol{F_3} = F_3\,(\boldsymbol{r_B}/\, r_B)$$

$$= 700\text{ N }[(-2\,\boldsymbol{i} - 3\,\boldsymbol{j} + 6\boldsymbol{k})/(2^2 + 3^2 + 6^2)^{1/2}]$$

$$= \{-200\,\boldsymbol{i} - 300\,\boldsymbol{j} + 600\,\boldsymbol{k}\}\text{ N}$$

EXAMPLE #1
(continued)

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$\Sigma F_x = -200 + F_X = 0$; solving gives $F_x = 200$ N

$\Sigma F_y = 400 - 300 + F_y = 0$; solving gives $F_y = -100$ N

$\Sigma F_z = -800 + 600 + F_z = 0$; solving gives $F_z = 200$ N

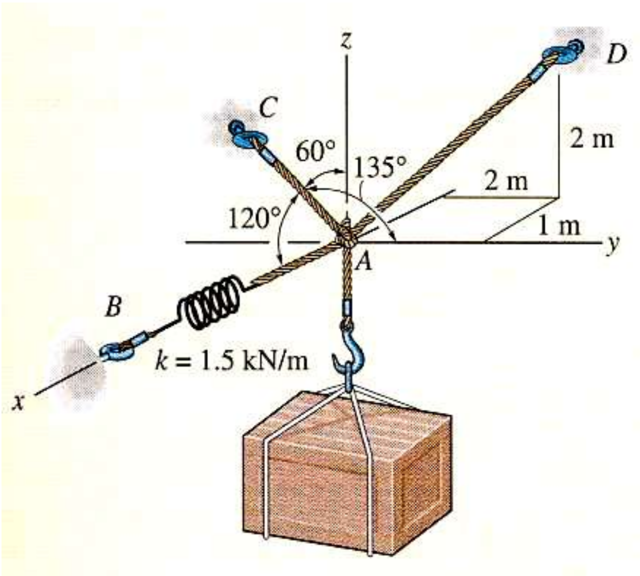
Thus, $\mathbf{F} = \{200\,\mathbf{i} - 100\,\mathbf{j} + 200\,\mathbf{k}\}$ N

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

EXAMPLE #2

Given: A 100 Kg crate, as shown, is supported by three cords. One cord has a spring in it.

Find: Tension in cords AC and AD and the stretch of the spring.



<u>Plan</u>:

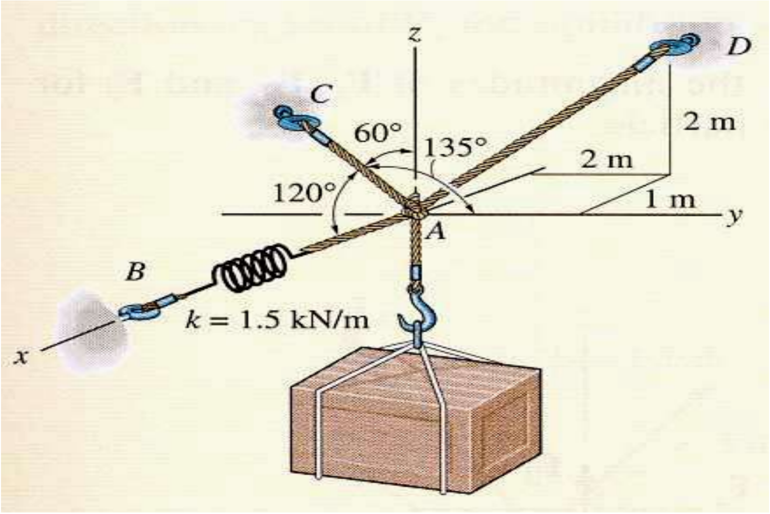
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

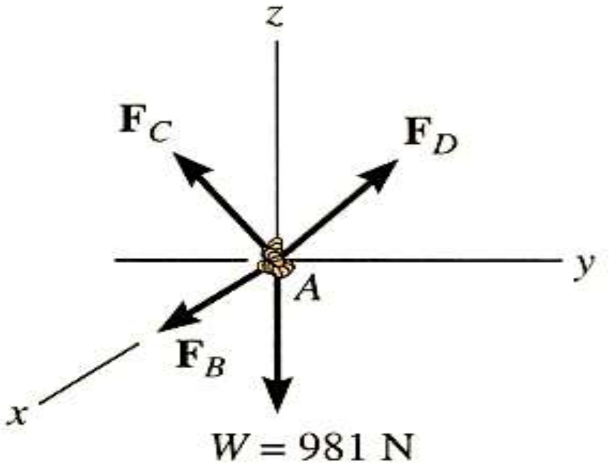
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

4) Find the spring stretch using $F_B = K * S$.

EXAMPLE #2 (continued)





FBD at A

$\boldsymbol{F_B}$ = F_B N $\boldsymbol{i}$

$\boldsymbol{F_C}$ = F_C N (cos 120° $\boldsymbol{i}$ + cos 135° $\boldsymbol{j}$ + cos 60° $\boldsymbol{k}$)

= {- 0.5 F_C $\boldsymbol{i}$ – 0.707 F_C $\boldsymbol{j}$ + 0.5 F_C $\boldsymbol{k}$} N

$\boldsymbol{F_D}$ = $F_D(\boldsymbol{r_{AD}}/r_{AD})$

= F_D N[(-1 $\boldsymbol{i}$ + 2 $\boldsymbol{j}$ + 2 $\boldsymbol{k}$)/(1^2 + 2^2 + 2^2)^{½}]

= {- 0.3333 F_D $\boldsymbol{i}$ + 0.667 F_D $\boldsymbol{j}$ + 0.667 F_D $\boldsymbol{k}$}N

EXAMPLE #2 (continued)

The weight is $\boldsymbol{W}$ = (- mg) $\boldsymbol{k}$ = (-100 kg * 9.81 m/sec^2) $\boldsymbol{k}$ = {- 981 $\boldsymbol{k}$} N

Now equate the respective $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ components to zero.

$$\sum F_x = F_B - 0.5F_C - 0.333F_D = 0$$

$$\sum F_y = -0.707\, F_C + 0.667\, F_D = 0$$

$$\sum F_z = 0.5\, F_C + 0.667\, F_D - 981\text{ N} = 0$$

Solving the three simultaneous equations yields

$F_C = 813$ N

$F_D = 862$ N

$F_B = 693.7$ N

The spring stretch is (from F = k * s)

$$s = F_B / k = 693.7\text{ N} / 1500\text{ N/m} = 0.462\text{ m}$$

Solving using Matrix Methods

If $\boldsymbol{AX} = \boldsymbol{B}$, $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, where $\boldsymbol{A}$, $\boldsymbol{X}$, and $\boldsymbol{B}$ are matrices.

Need to create solving structure

$$F_B - 0.5F_C - 0.333F_D = 0$$

$$0.707\,F_C + 0.667\,F_D = 0$$

$$0.5\,F_C + 0.667\,F_D - 981 = 0$$

$$F_B - 0.5\,F_C - 0.333\,F_D = 0$$

$$0\,F_B + 0.707\,F_C + 0.667\,F_D = 0$$

$$0\,F_B + 0.5\,F_C + 0.667\,F_D = 981$$

$$\begin{bmatrix} 1 & -0.5 & -0.333 \\ 0 & 0.707 & 0.667 \\ 0 & 0.5 & 0.667 \end{bmatrix} \begin{bmatrix} F_B \\ F_C \\ F_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 981 \end{bmatrix}$$

F_B	F_C	F_D	
1	-0.5	-0.333	0
0	0.707	0.667	0
0	0.5	0.667	981

CONCEPT QUIZ

1. In 3-D, when you know the direction of a force but not its magnitude, how many unknowns corresponding to that force remain?

A) One B) Two C) Three D) Four

2. If a particle has 3-D forces acting on it and is in static equilibrium, the components of the resultant force (ΣF_x, ΣF_y, and ΣF_z) ___ .

A) have to sum to zero, e.g., -5 ***i*** + 3 ***j*** + 2 ***k***

B) have to equal zero, e.g., 0 ***i*** + 0 ***j*** + 0 ***k***

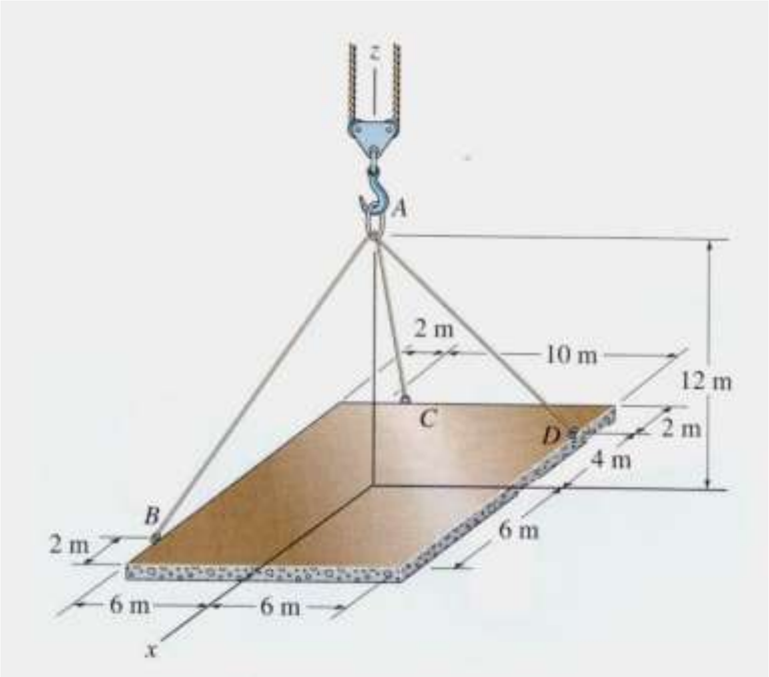
C) have to be positive, e.g., 5 ***i*** + 5 ***j*** + 5 ***k***

D) have to be negative, e.g., -5 ***i*** - 5 ***j*** - 5 ***k***

GROUP PROBLEM SOLVING

Given: A 150 Kg plate, as shown, is supported by three cables and is in equilibrium.

Find: Tension in each of the cables.



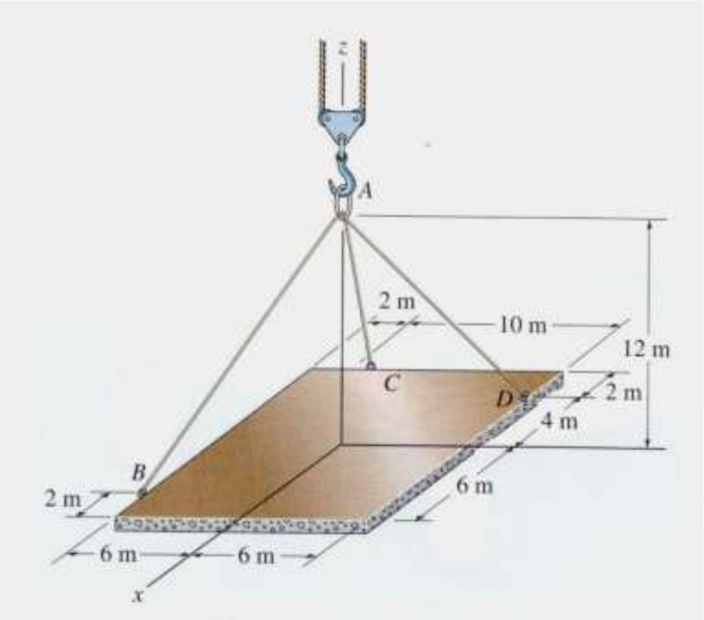
<u>Plan</u>:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

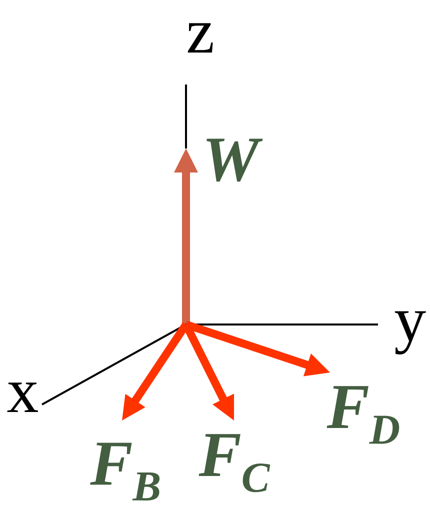
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

GROUP PROBLEM SOLVING (continued)



FBD of Point A:



$\boldsymbol{W}$ = load or weight of plate = (mass)(gravity)

= 150 (9.81) $\boldsymbol{k}$ = 1472 $\boldsymbol{k}$ N

$\boldsymbol{F_B}$ = $F_B(\boldsymbol{r_{AB}}/r_{AB})$ = F_B N (4 $\boldsymbol{i}$ – 6 $\boldsymbol{j}$ – 12 $\boldsymbol{k}$)m/(14 m)

$\boldsymbol{F_C}$ = $F_C(\boldsymbol{r_{AC}}/r_{AC})$ = F_C(-6 $\boldsymbol{i}$ – 4 $\boldsymbol{j}$ – 12 $\boldsymbol{k}$)m/(14 m)

$\boldsymbol{F_D}$ = $F_D(\boldsymbol{r_{AD}}/r_{AD})$ = F_D(-4 $\boldsymbol{i}$ + 6 $\boldsymbol{j}$ – 12 $\boldsymbol{k}$)m/(14 m)

GROUP PROBLEM SOLVING (continued)

The particle A is in equilibrium, hence

$\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$$\sum F_x = (4/14)F_B - (6/14)F_C - (4/14)F_D = 0$$

$$\sum F_y = (-6/14)F_B - (4/14)F_C + (6/14)F_D = 0$$

$$\sum F_z = (-12/14)F_B - (12/14)F_C - (12/14)F_D + 1472 = 0$$

Solving the three simultaneous equations gives

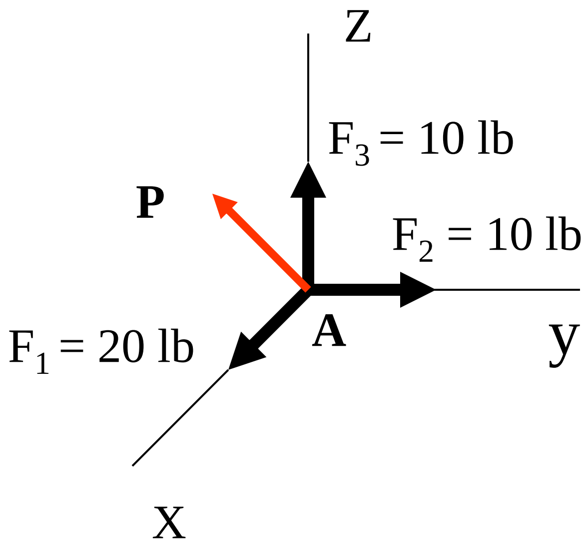
$F_B = 858 \text{ N}$

$F_C = 0 \text{ N}$

$F_D = 858 \text{ N}$

QUIZ

1. Four forces act at point A and point A is in equilibrium. Select the correct force vector ***P***.



   A) {-20 ***i*** + 10 ***j*** – 10 ***k***}lb

   B) {-10 ***i*** – 20 ***j*** – 10 ***k***} lb

   C) {+ 20 ***i*** – 10 ***j*** – 10 ***k***}lb

   D) None of the above.

2. In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

   A) One B) Two C) Three D) Four