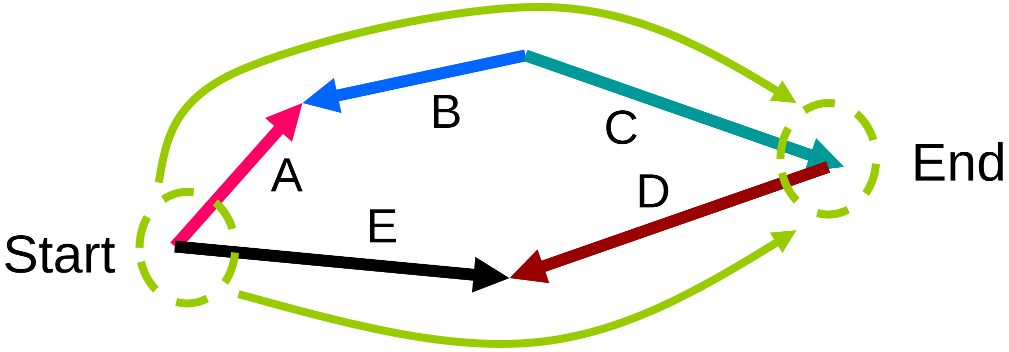
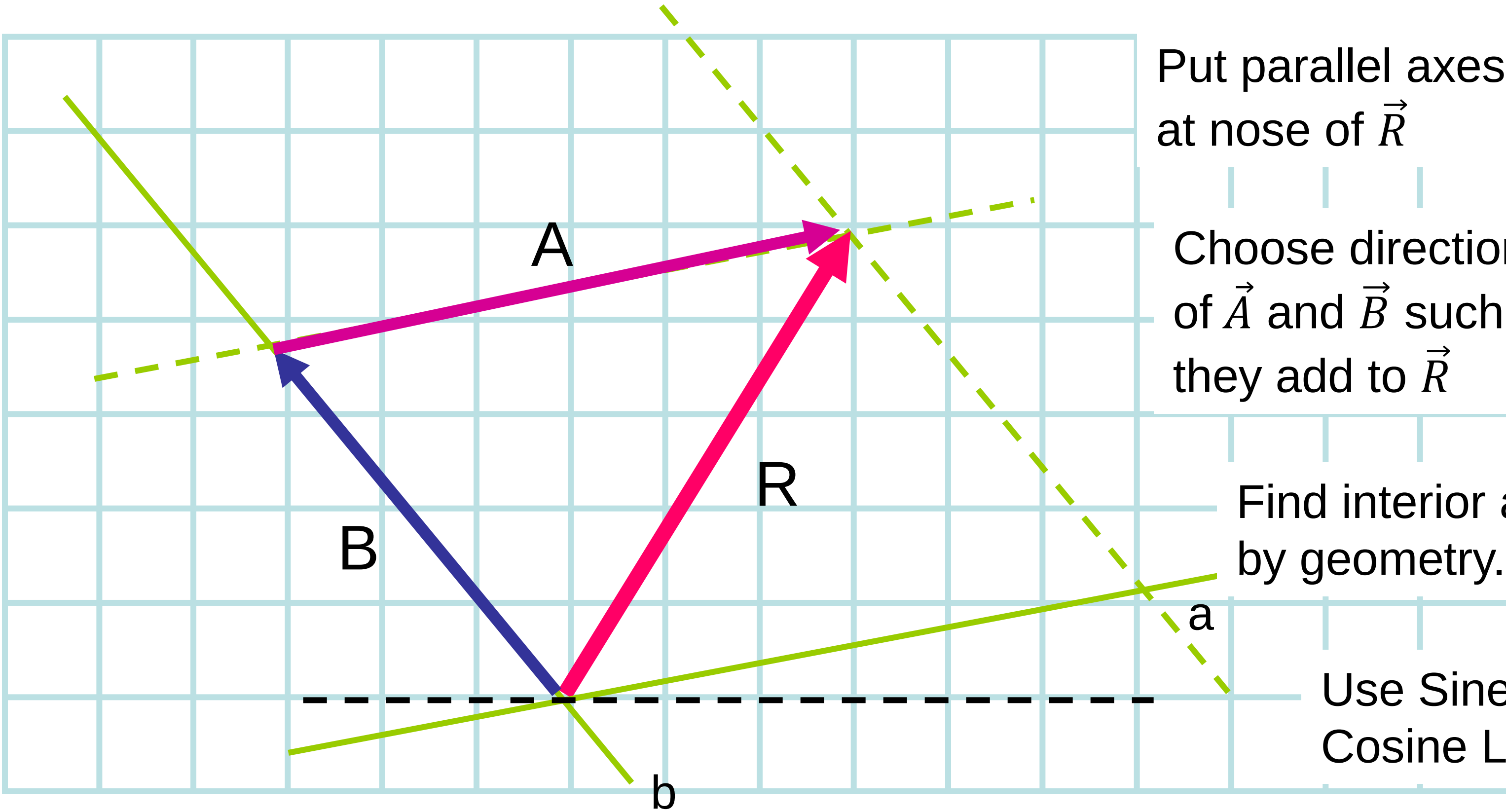
Vector Diagrams & Equations

- Pick any start and end points
- Following an arrow in same direction is +
- Follow in opposite direction is –



$$\vec{A} - \vec{B} + \vec{C} = \vec{E} - \vec{D}$$

Resolve $\vec{R}$ *along given axes such that* $\vec{R} = \vec{A} + \vec{B}$



Put parallel axes at nose of $\vec{R}$

Choose directions of $\vec{A}$ and $\vec{B}$ such that they add to $\vec{R}$

Find interior angles by geometry.

Use Sine and Cosine Laws

$$Sine\ \ Law: \quad \frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

$$Cosine\ \ Law: \quad C^2 = A^2 + B^2 - 2AB\cos\gamma$$

Vector Forms

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{A} = A, \angle\alpha, \angle\beta, \angle\gamma$$

$$\vec{A} = A\hat{u}_A$$

Converting between forms

$$\hat{u}_A = \frac{\vec{A}}{A} \qquad A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

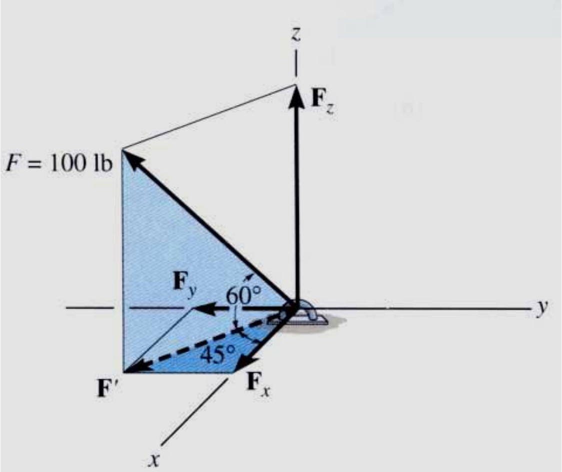
$$\hat{u}_A = \frac{A_x}{A}\hat{\imath} + \frac{A_y}{A}\hat{\jmath} + \frac{A_z}{A}\hat{k}$$

$$cos\alpha = \frac{A_x}{A} \qquad cos\beta = \frac{A_y}{A} \qquad cos\gamma = \frac{A_z}{A}$$

$$\hat{u}_a = cos\alpha\,\hat{\imath} + cos\beta\,\hat{\jmath} + cos\gamma\,\hat{k}$$

$$1 = cos^2\alpha + cos^2\beta + cos^2\gamma$$

If we have angle information can write a vector in Cartesian form.



$F_z = 100 \sin 60° = 86.60$ lb

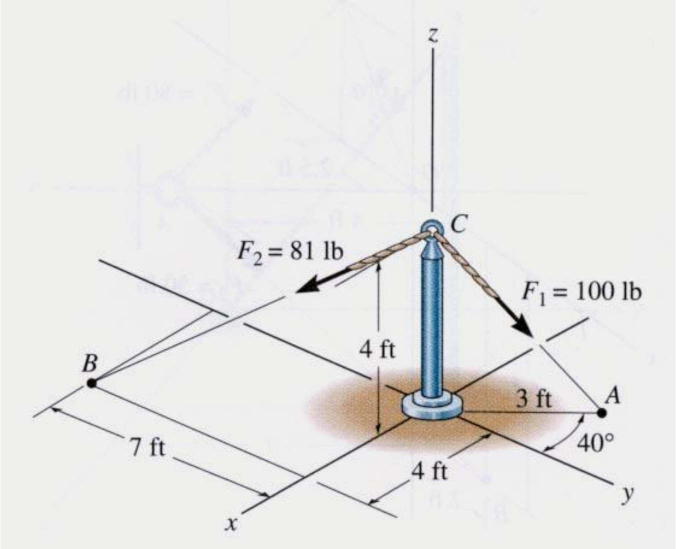
$F' = 100 \cos 60° = 50.00$ lb

$F_x = 50 \cos 45° = 35.36$ lb

$F_y = 50 \sin 45° = 35.36$ lb

$\boldsymbol{F} = \{35.36\, \boldsymbol{i} - 35.36\, \boldsymbol{j} + 86.60\, \boldsymbol{k}\}$ lb

If we have positional information can write a vector in Cartesian form.

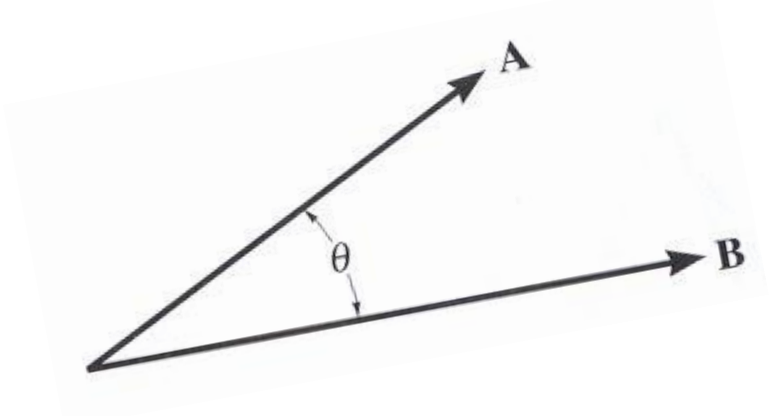


$\boldsymbol{F_2} = (81 \text{ lb})\{\boldsymbol{r_{CB}}/r_{CB}\}$

$\boldsymbol{F_2} = (81 \text{ lb})\,(4\, \boldsymbol{i} - 7\, \boldsymbol{j} - 4\, \boldsymbol{k})/9$

$\boldsymbol{F_2} = \{36\, \boldsymbol{i} - 63\, \boldsymbol{j} - 36\, \boldsymbol{k}\}$ lb

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



If we know two vectors in Cartesian form, finding θ is easy since we have two methods of doing the dot product; $\vec{A} \bullet \vec{B}$ = A B cosθ & $\vec{A} \bullet \vec{B}$ = $A_x B_x + A_y B_y + A_z B_z$

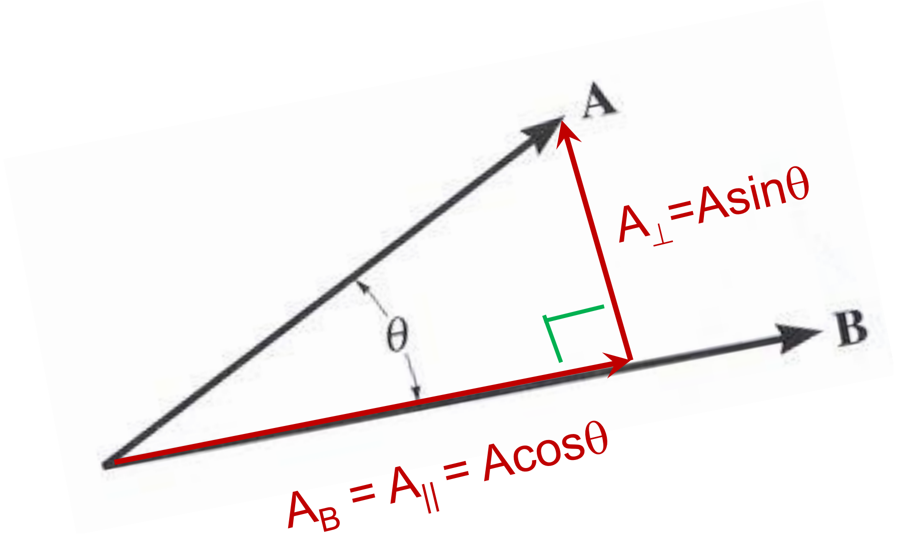
$$AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\cos\theta = \frac{A_x B_x + A_y B_y + A_z B_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

More usually just written $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{A\,B}$ or $\cos\theta = \hat{u}_A \cdot \hat{u}_B$

Projection

How much of $\vec{A}$ is in the same direction as $\vec{B}$



$$\vec{A}_B = A_B \hat{u}_B$$

$$\vec{A}_B = (\vec{A} \cdot \hat{u}_B)\hat{u}_B$$

$$\vec{A}_\perp = \vec{A} - \vec{A}_B$$