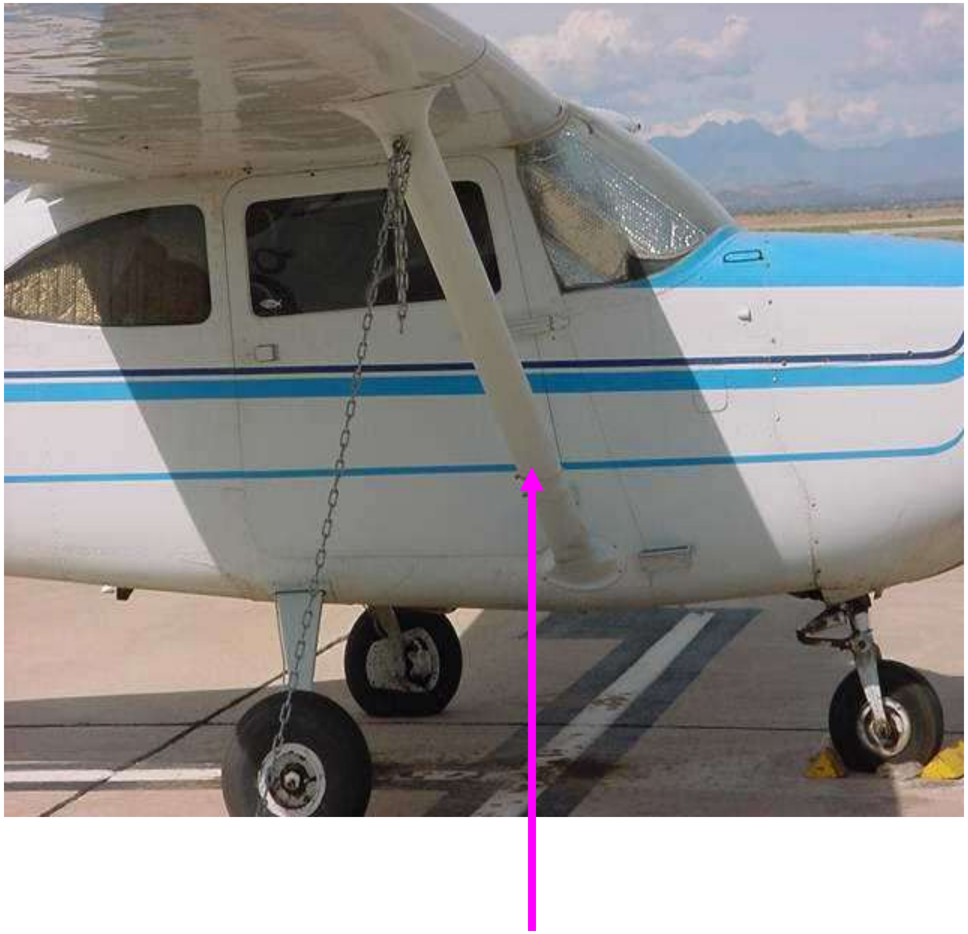
APPLICATIONS



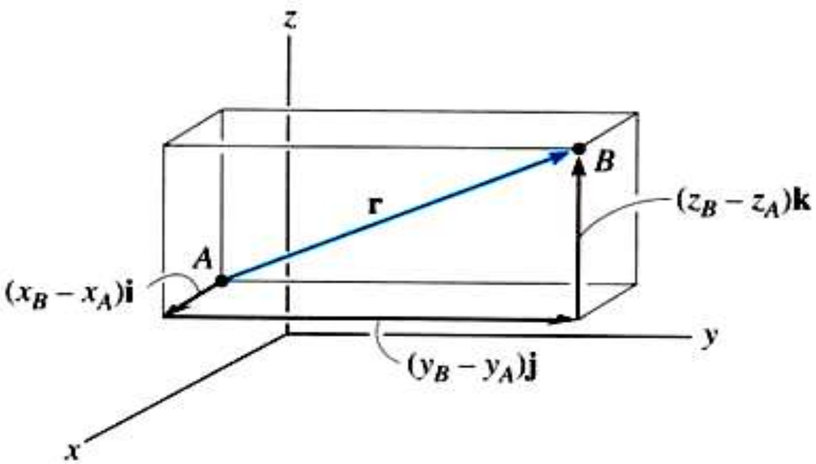
Wing strut

How can we represent the force along the wing strut in a 3-D Cartesian vector form?

POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.

Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

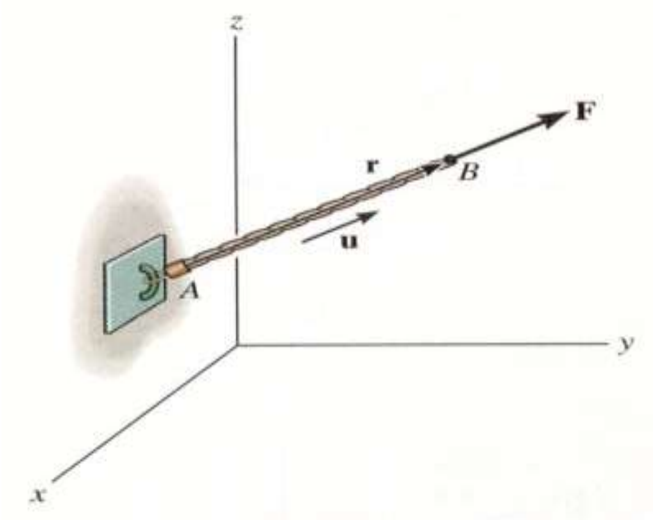


The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\boldsymbol{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point. So ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE
(Section 2.8)



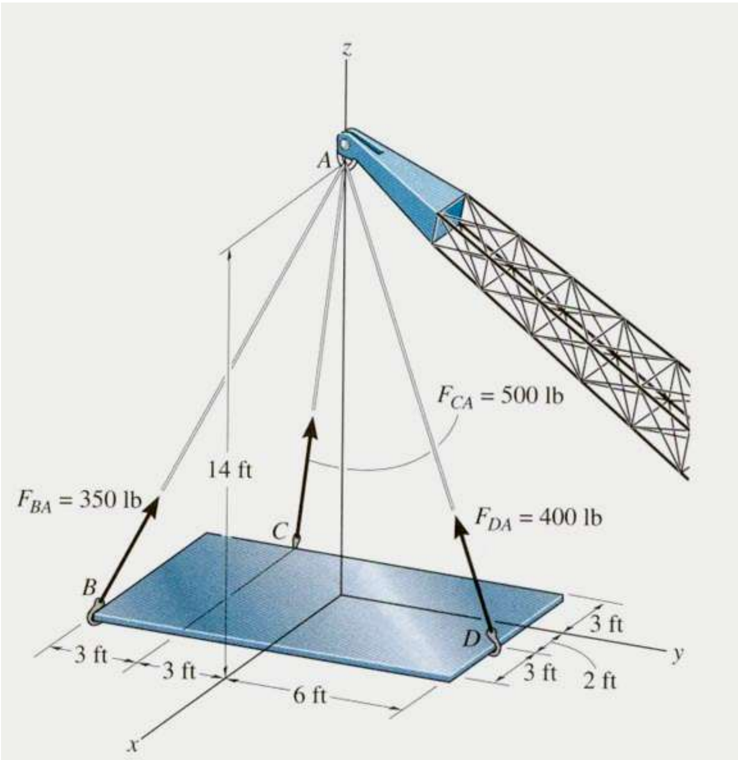
If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\,\boldsymbol{u}_{AB}$.

EXAMPLE



Given: 400 lb force along the cable DA.

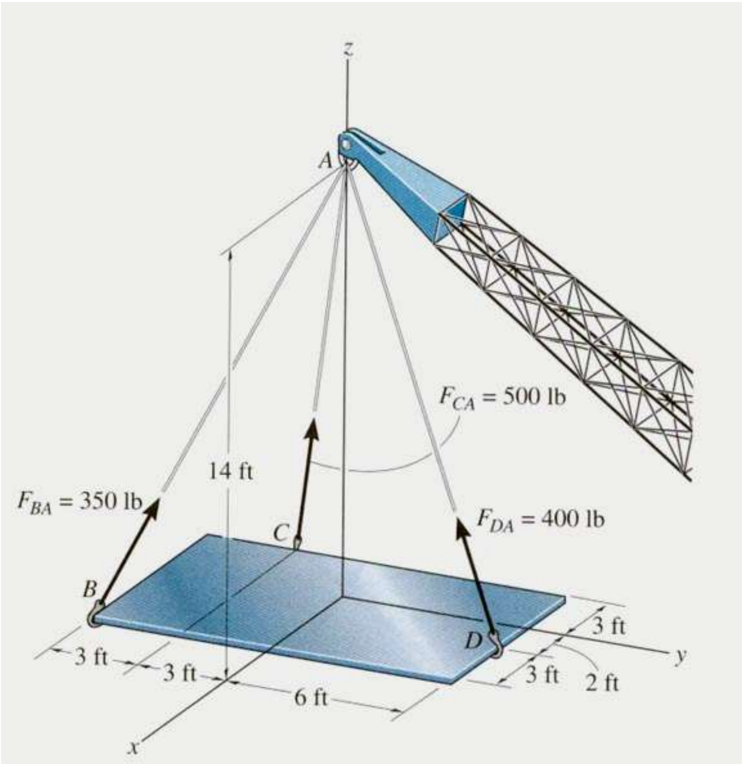
Find: The force $\boldsymbol{F_{DA}}$ in the Cartesian vector form.

<u>Plan</u>:

- Find the position vector $\boldsymbol{r_{DA}}$ and the unit vector $\boldsymbol{u_{DA}}$.

2. Obtain the force vector as $\boldsymbol{F_{DA}}$ = 400 lb $\boldsymbol{u_{DA}}$.

EXAMPLE (continued)



The figure shows that when relating D to A, we will have to go -2 ft in the x-direction, -6 ft in the y-direction, and +14 ft in the z-direction. Hence,

$\mathbf{r}_{DA} = \{-2\,\mathbf{i} - 6\,\mathbf{j} + 14\,\mathbf{k}\}$ ft.

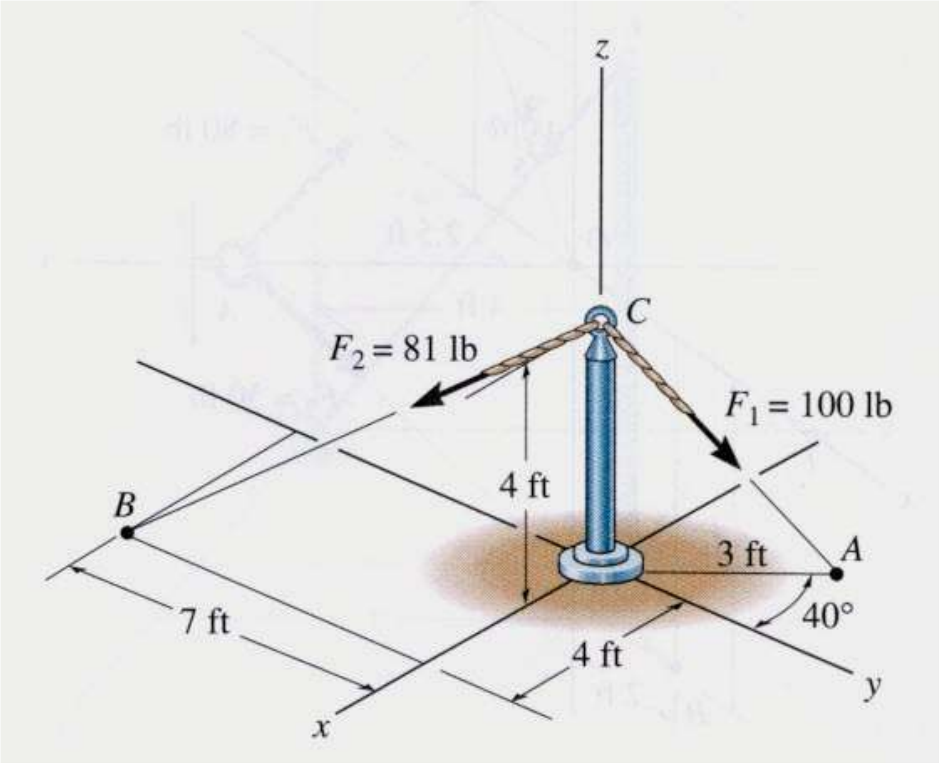
We can also find $\mathbf{r}_{DA}$ by subtracting the coordinates of D from the coordinates of A.

$$r_{DA} = (2^2 + 6^2 + 14^2)^{0.5} = 15.36 \text{ ft}$$

$\mathbf{u}_{DA} = \mathbf{r}_{DA}/r_{DA}$ and $\mathbf{F}_{DA} = 400\,\mathbf{u}_{DA}$ lb

$$\mathbf{F}_{DA} = 400\{(-2\,\mathbf{i} - 6\,\mathbf{j} + 14\,\mathbf{k})/15.36\} \text{ lb}$$

$$= \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\} \text{ lb}$$



EXAMPLE

Given: Two forces are acting on a pipe as shown in the figure.

Find: The magnitude and the coordinate direction angles of the resultant force.

<u>Plan</u>:

1) Find the forces along CA and CB in the Cartesian vector form.

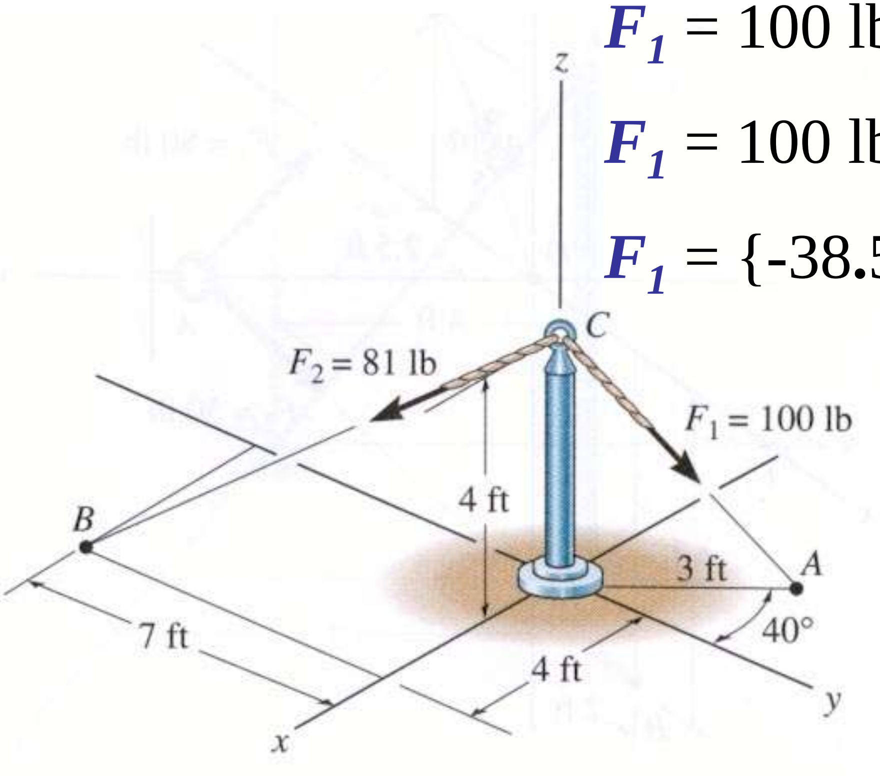
2) Add the two forces to get the resultant force, $\boldsymbol{F_R}$.

3) Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

$$\boldsymbol{F_1} = 100 \text{ lb}\{\boldsymbol{r_{CA}}/r_{CA}\}$$

$$\boldsymbol{F_1} = 100 \text{ lb}(-3 \sin 40° \, \boldsymbol{i} + 3 \cos 40° \, \boldsymbol{j} - 4 \, \boldsymbol{k})/5$$

$$\boldsymbol{F_1} = \{-38.57 \, \boldsymbol{i} + 45.96 \, \boldsymbol{j} - 80 \, \boldsymbol{k}\} \text{ lb}$$



$$\boldsymbol{F_2} = 81 \text{ lb}\{\boldsymbol{r_{CB}}/r_{CB}\}$$

$$\boldsymbol{F_2} = 81 \text{ lb}(4 \, \boldsymbol{i} - 7 \, \boldsymbol{j} - 4 \, \boldsymbol{k})/9$$

$$\boldsymbol{F_2} = \{36 \, \boldsymbol{i} - 63 \, \boldsymbol{j} - 36 \, \boldsymbol{k}\} \text{ lb}$$

$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} = \{-2.57 \, \boldsymbol{i} - 17.04 \, \boldsymbol{j} - 116 \, \boldsymbol{k}\} \text{ lb}$$

$$F_R = (2.57^2 + 17.04^2 + 116^2) = 117.3 \text{ lb} = 117 \text{ lb}$$

$$\alpha = \cos^{-1}(-2.57/117.3) = 91.3°, \quad \beta = \cos^{-1}(-17.04/117.3) = 98.4°$$

$$\gamma = \cos^{-1}(-116/117.3) = 172°$$