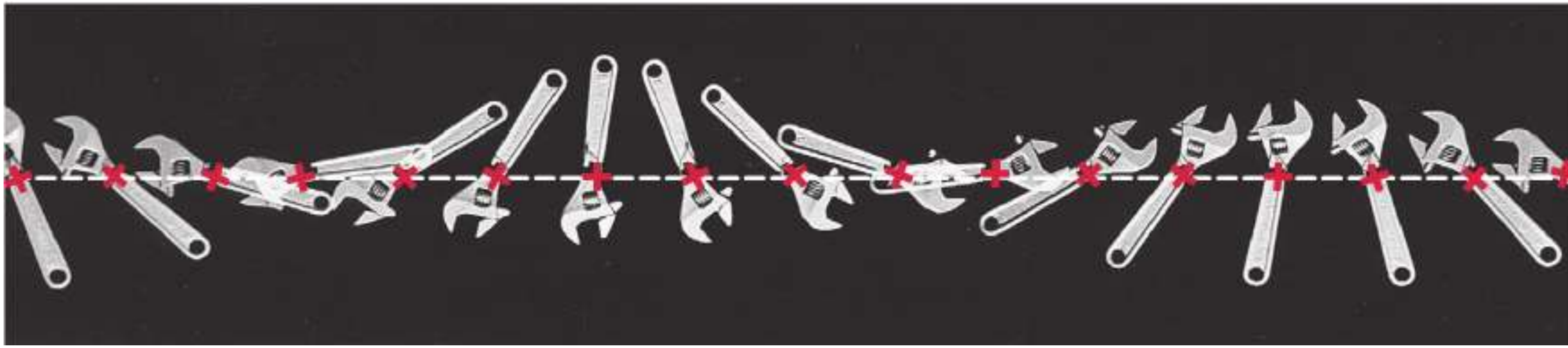


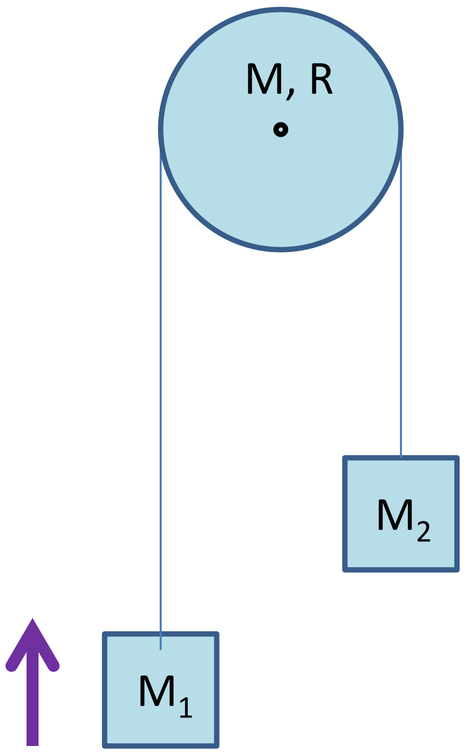
- Motion broken into
  - Motion of CM $\sum F_x = ma_x \, , \sum F_y = ma_y$
  - Motion about CM $\sum \tau_{CM} = I_{CM} \alpha_{CM}$

Two Problem Types

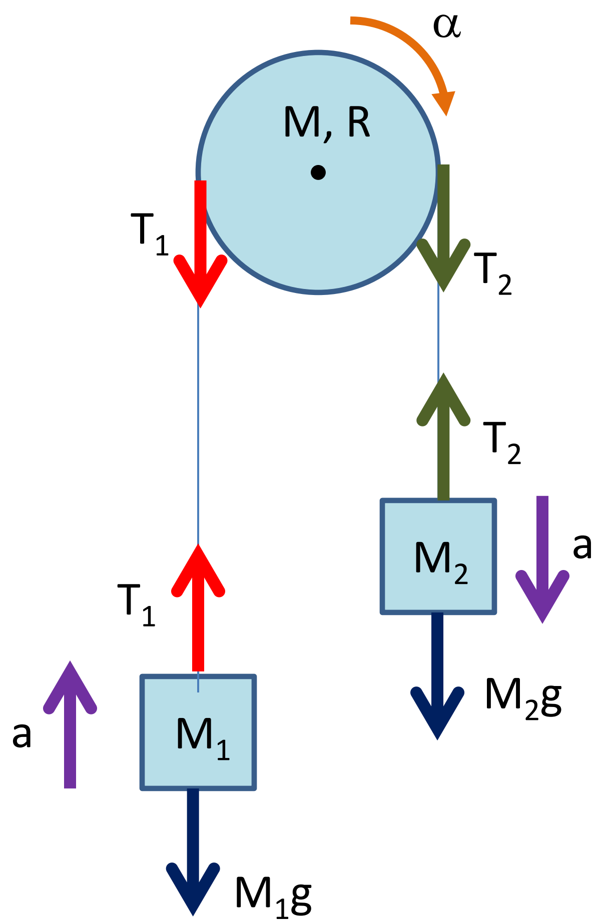
- Real Pulleys

- Rolling

1. Real Fixed Pulleys



- Fixed pulley, no *a*. Interested only in τ's and α. TFBD only
- Friction btw string and pulley. Tension varies over pulley.
- Treat as two different T's
- String relates a_{block} to α and R
- R is lever arm for T's (T ⊥ R always)



$$RT_1 - RT_2 = -I\alpha$$

$$I = \tfrac{1}{2}MR^2 \quad \& \quad \alpha = a/R$$

$$T_1 - T_2 = -\tfrac{1}{2}Ma$$

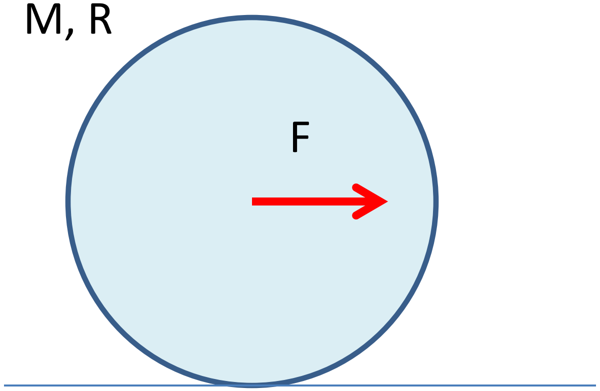
$$T_1 - M_1g = +M_1a$$

$$T_2 - M_2g = -M_2a$$

$$a = (M_2 - M_1)g/(M_1 + M_2 + \tfrac{1}{2}M)$$

2. Rolling Without Slipping

Disk
M, R

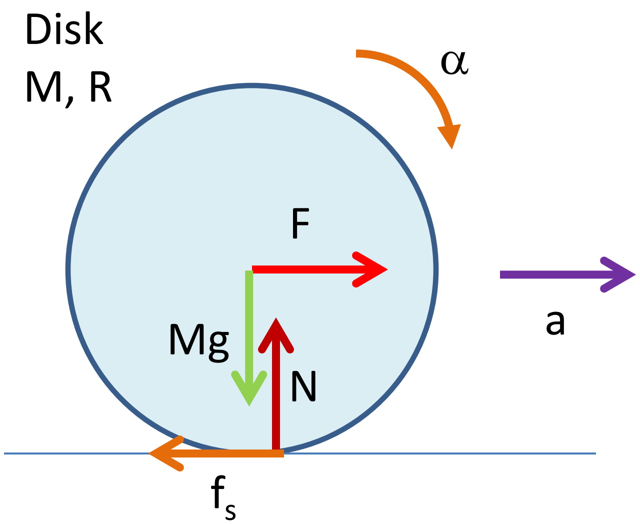


Moving right & speeding up

Given μ_s and μ_k. Find a and f_s.

If F is too big , disk will slip.
What is F_{slip}?

- a and α dir^{n}s related
- $a = R\alpha$
- f_s, not f_k , not f_s^{max}
- Must find formula for f_s
- Dir^n of f_s must be consistent with Newton's Laws
- f_s and a must be expressed in terms of given quantities



Find consistent a & α

Add all forces but f_s

F, N, & W cannot produce a torque. Some “other” force must!

$$-Rf_s = -I\alpha$$

$$I = \tfrac{1}{2}MR^2 \;\;\&\;\; \alpha = a/R \;\Rightarrow\; f_s = \tfrac{1}{2}Ma$$

$$F - f_s = Ma \quad \& \quad N - Mg = 0$$

$$F - \tfrac{1}{2}Ma = Ma \;\Rightarrow\; a = 2F/3M$$

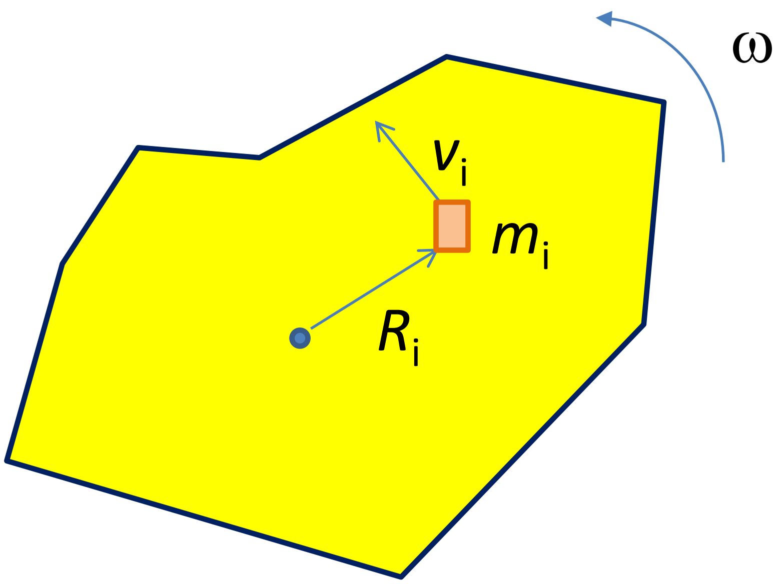
$$f_s = \tfrac{1}{2}Ma \;\Rightarrow\; f_s = F/3$$

Will start to slip …

- Note $f_s = F/3$ so $f_s \uparrow$ when $F \uparrow$
- But f_s cannot exceed f_s^{max}
- $f_s^{max} = \mu_s N = \mu_s Mg$
- So will slip when $f_s = f_s^{max}$ or
- $F/3 = \mu_s Mg$ or $F = 3\mu_s Mg$

Kinetic Energy & Rotation

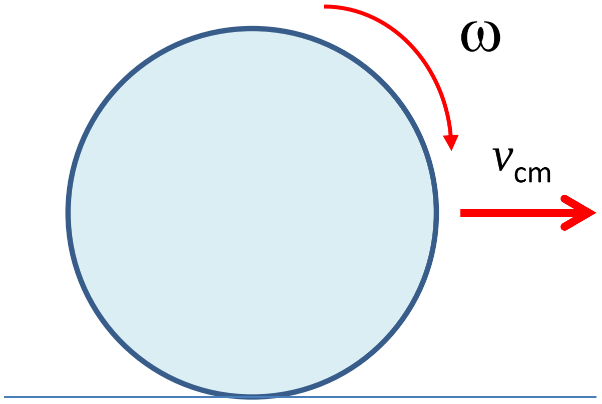
- A rotating object has no linear KE since v = 0
- But each piece is moving $K_i = \frac{1}{2} m_i v_i^2$



$$
\begin{aligned}
K_{total} &= \sum K_i \\
&= \sum \tfrac{1}{2} m_i v_i^2 \\
&= \frac{1}{2} \sum m_i (R_i \omega)^2 \\
&= \frac{1}{2} \left(\sum m_i R_i^2 \right) \omega^2 \\
&= \tfrac{1}{2} I \omega^2
\end{aligned}
$$

Kinetic Energy & Rolling

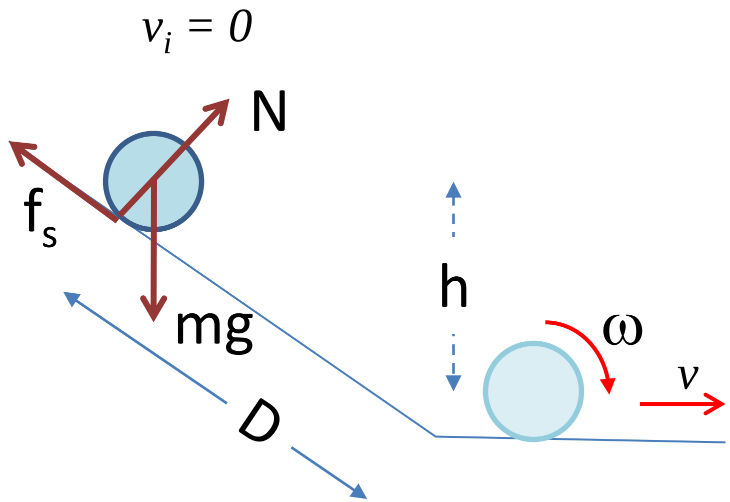
- Object has linear and rotational KE



- $K_{rolling} = \frac{1}{2} m_i v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$

- If no slipping, $\omega = v_{CM}/R$

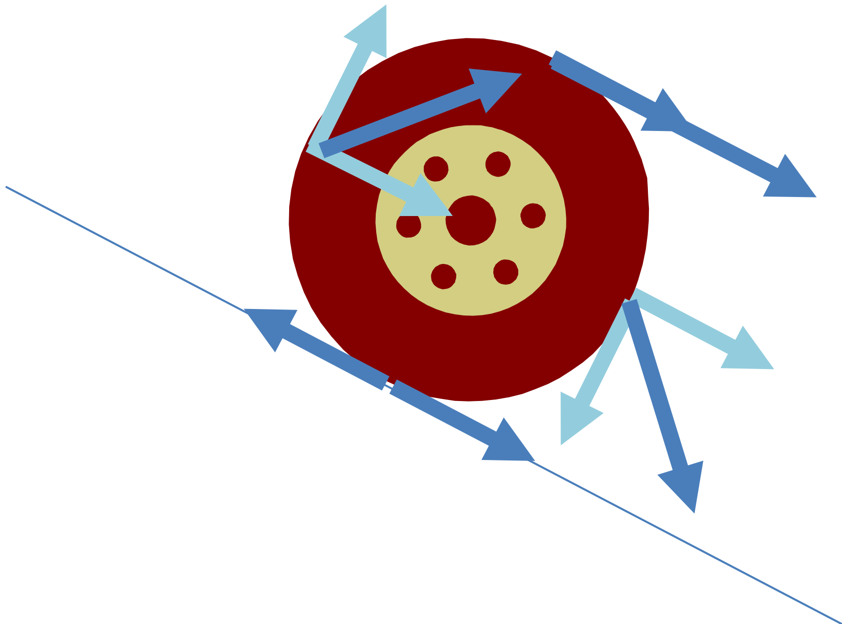
Example



$$W = -mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

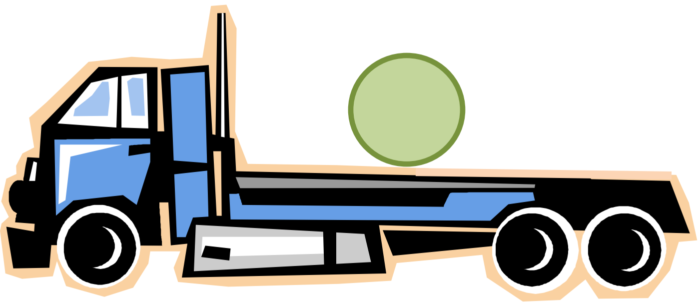
- Do any of the forces do work?
- No!!!!
- N is ⊥ to D, mg included in PE

Rolling and W_{fs}



- Contact is instantaneously at rest, D = 0 for f_s
- W_{fs} = 0 as long as surface is not moving!

Rolling and W_{fs}



- When truck moves gently forward, unsecured barrel will roll backwards off truck.
- Here $K_{rolling}$ increases because static friction does positive work. Here f_s acts over the distance truck moves.
- Won't do these cases.