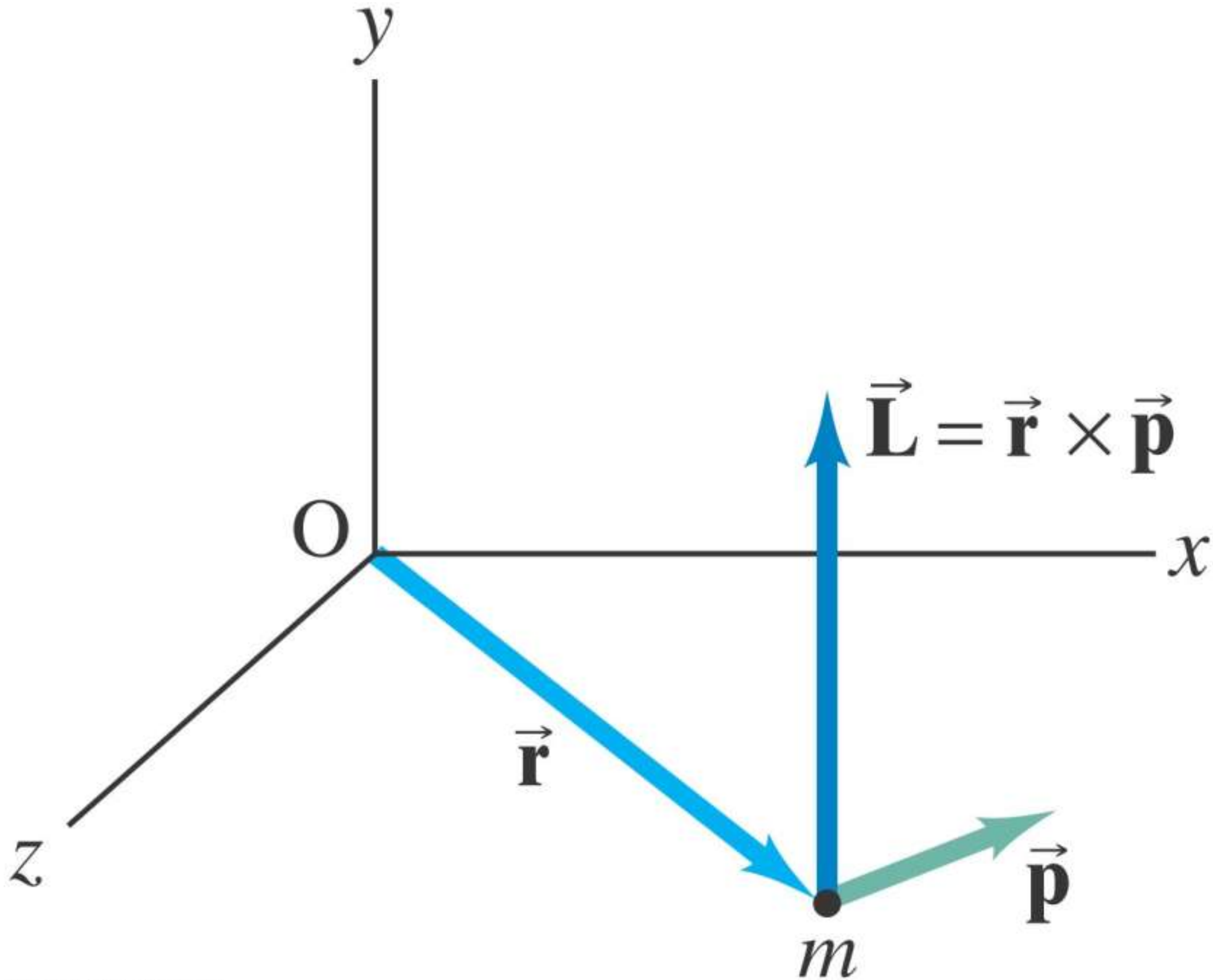
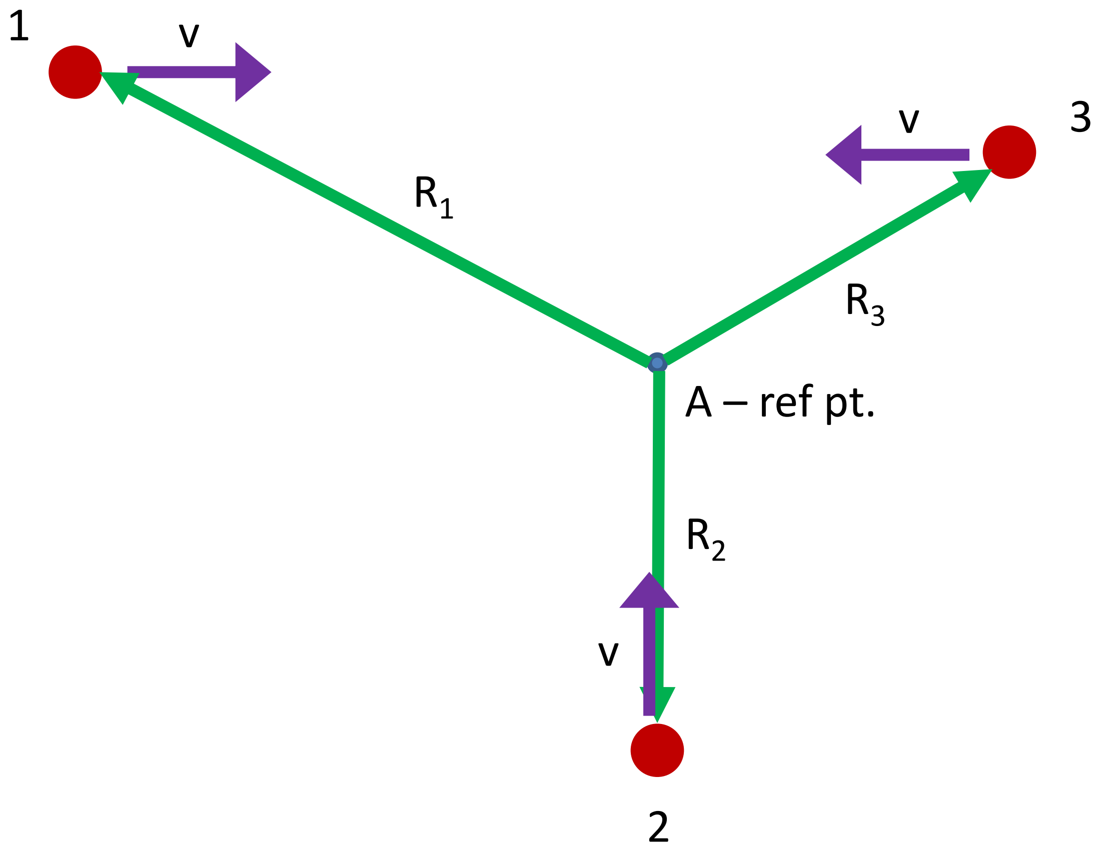

y
$\vec{L} = \vec{r} \times \vec{p}$
O
x
$\vec{r}$
z
$\vec{p}$
m


1
v
3
v
R1
R3
A – ref pt.
R2
v
2

$$\vec{L} = \vec{R} \times \vec{P}$$

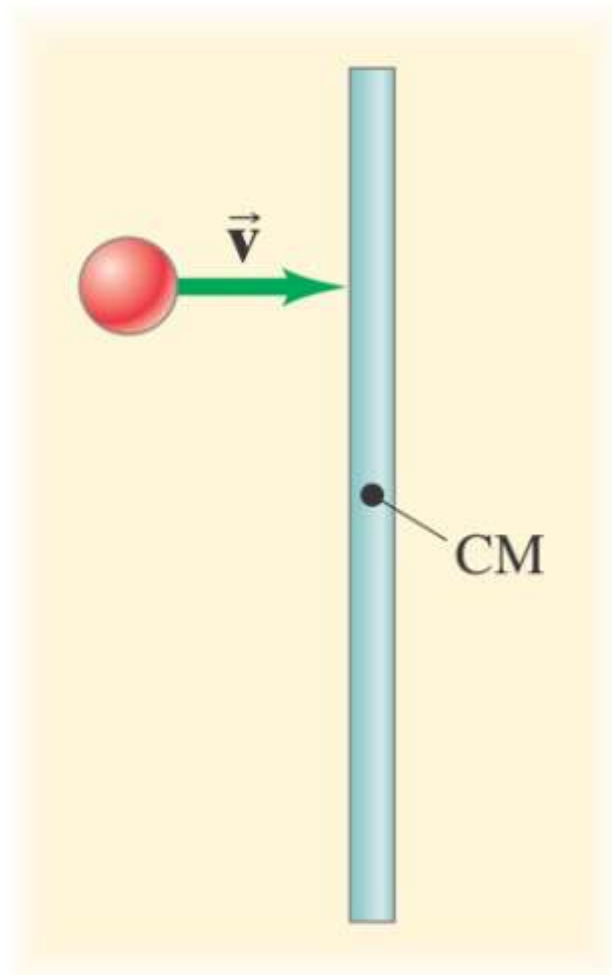
$$\frac{d\vec{L}}{dt} = \frac{d\vec{R}}{dt} \times \vec{P} + \vec{R} \times \frac{d\vec{P}}{dt}$$

$$\frac{d\vec{L}}{dt} = \vec{v} \times m\vec{v} + \vec{R} \times m\frac{d\vec{v}}{dt}$$

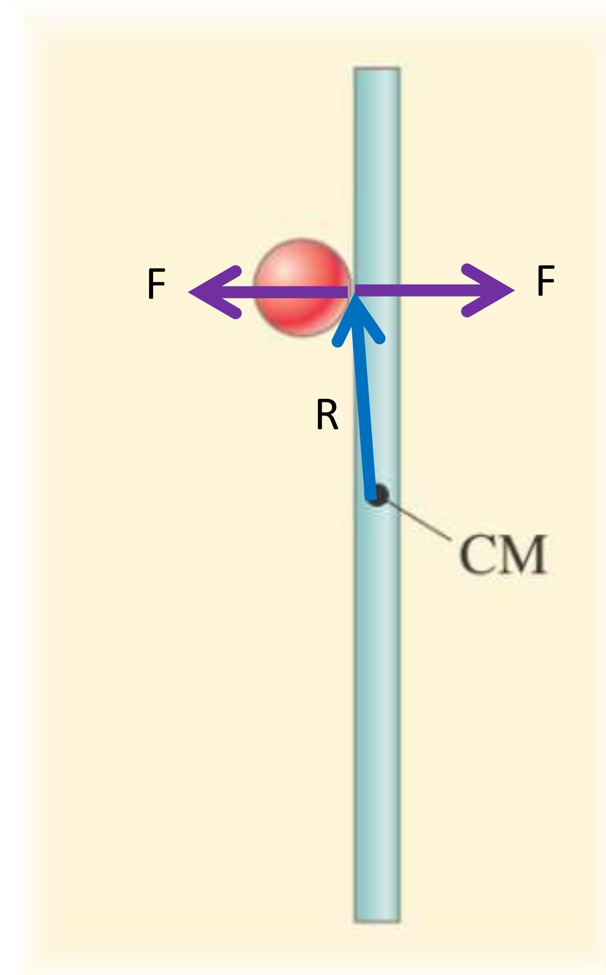
$$\frac{d\vec{L}}{dt} = 0 + \vec{R} \times m\vec{a}$$

$$\frac{d\vec{L}}{dt} = \vec{R} \times \vec{F}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau}$$








Torques will be equal and opposite!

$$\overrightarrow{\tau_1} = \frac{d\overrightarrow{L_1}}{dt}$$

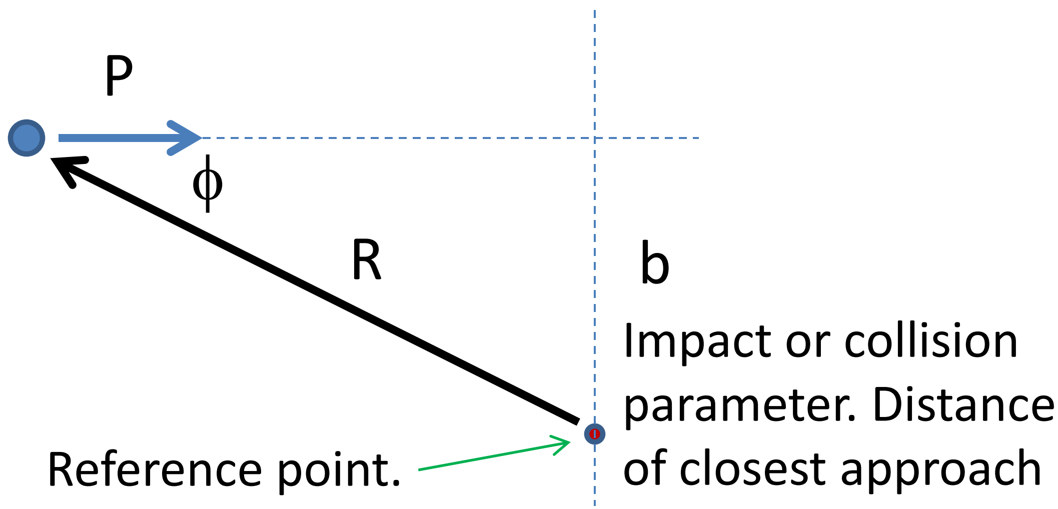
$$\overrightarrow{\tau_2} = \frac{d\overrightarrow{L_2}}{dt}$$

$$0 = \frac{d}{dt}(\overrightarrow{L_1}+\overrightarrow{L_2})$$

$$\overrightarrow{L_f} = \overrightarrow{L_i}$$

Law of Conservation of Angular Momentum

# Special Cases
# 1: Particle Travelling in a Line

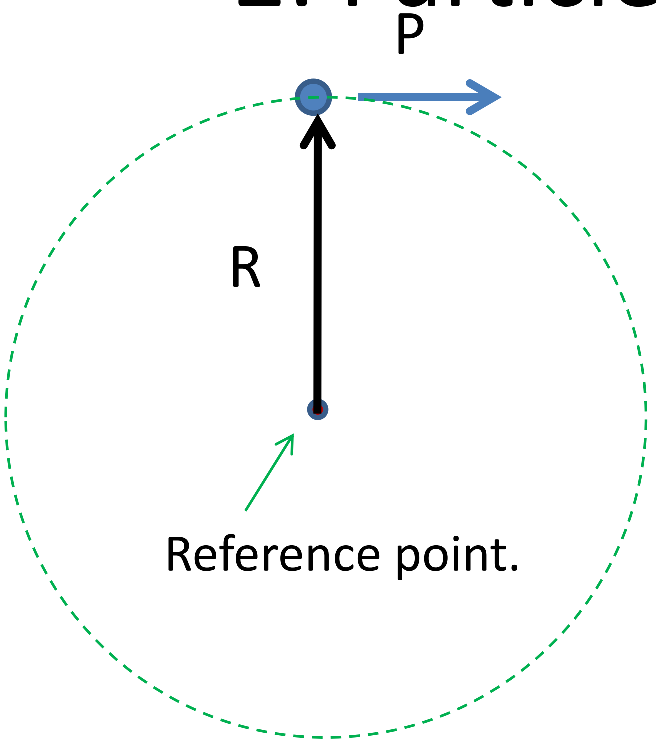



$L = Rp\sin\phi = bp$

If object is not a particle can treat as a particle if size $r \ll b$ and if $I\omega \ll bp$.

# Special Cases

# 2: Particle Travelling in a Circle

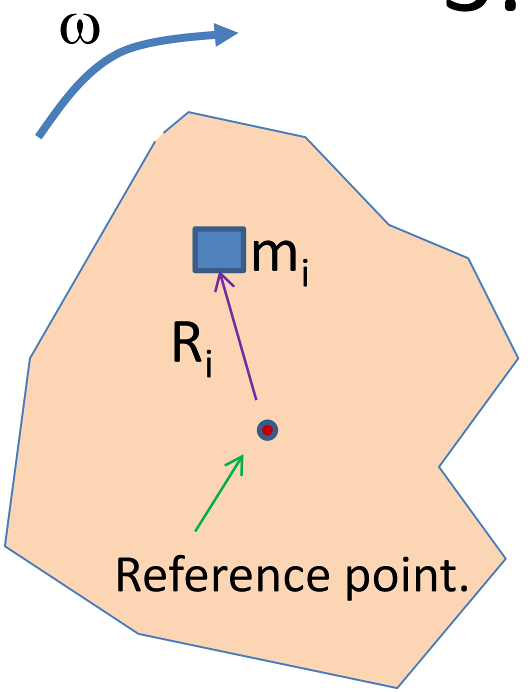



$$L = Rp = Rmv = R^2m\omega$$

"Orbital" Angular Momentum

If object is not a particle can treat as a particle if size $r \ll R$ and if $I\omega \ll bp$.

# Special Cases
# 3: Object Spinning



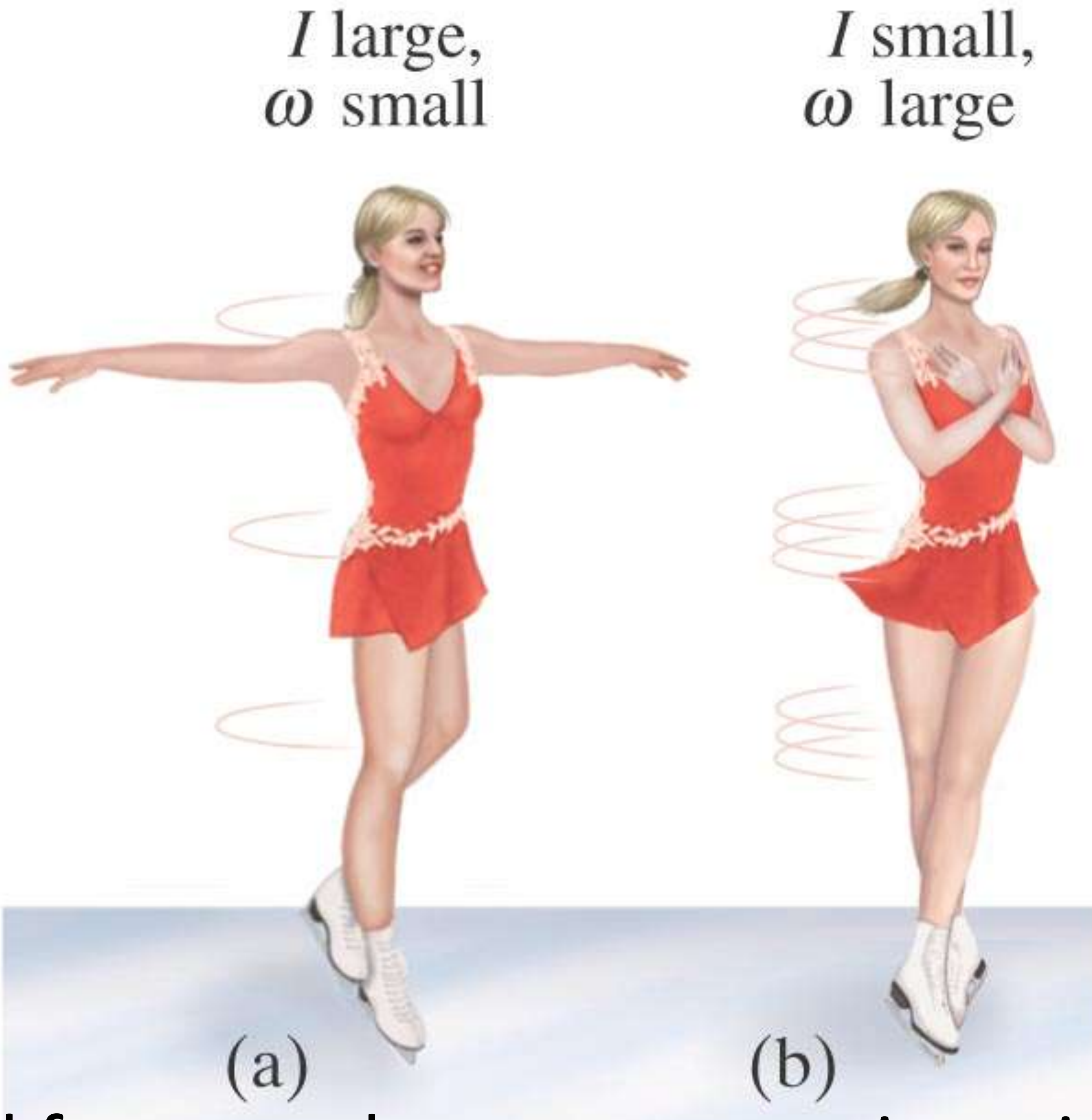

Consider one particle. It is rotating in a circle.

$$\therefore L_i = m_i R_i^2 \omega$$

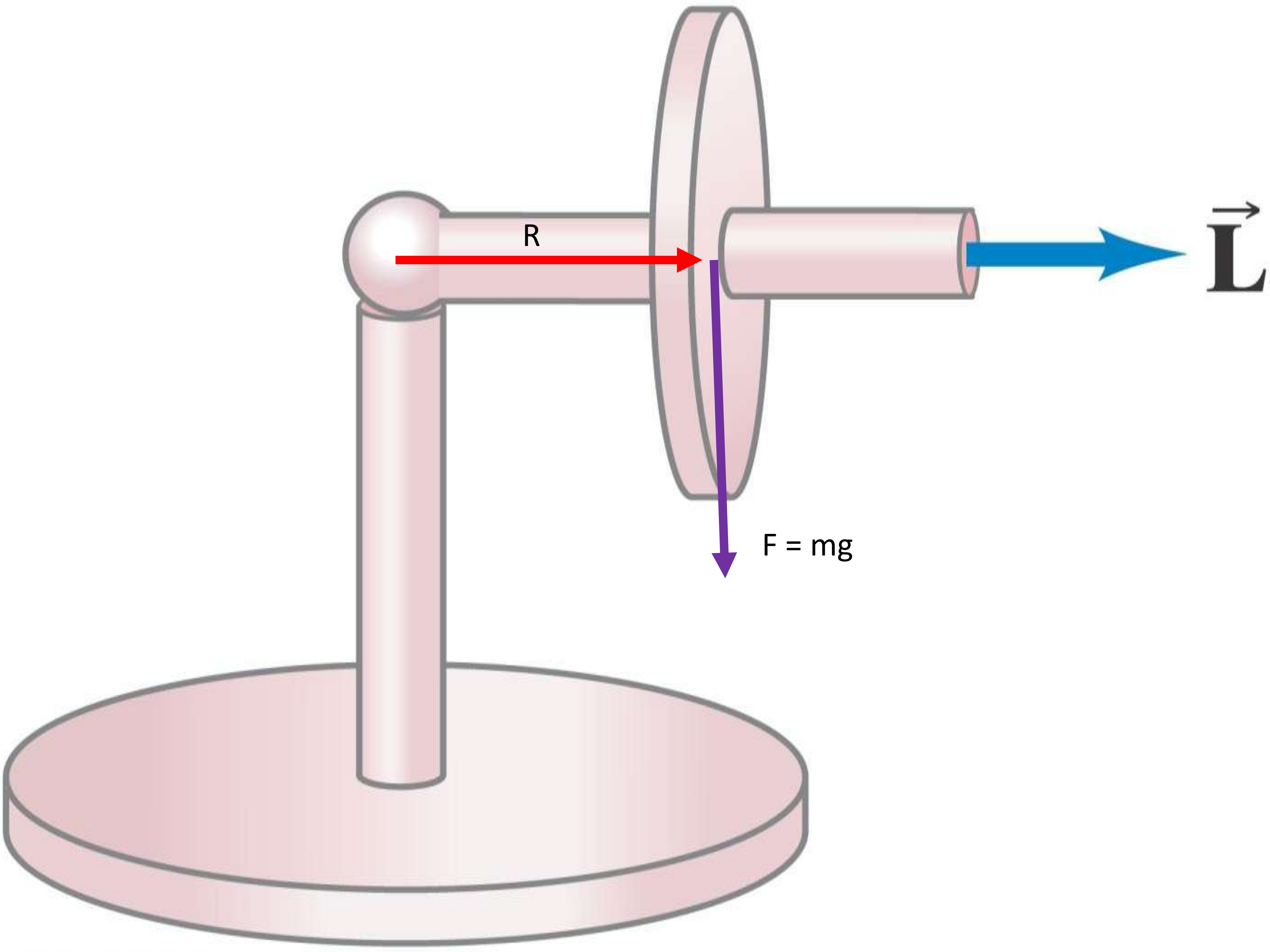
For total L add up all the contributions.

$$L = \Sigma m_i R_i^2 \omega = I\omega$$

"Spin" Angular Momentum




Internal forces and torques occur in action-reaction pairs. Angular Momentum is conserved.


R
F = mg
$\vec{L}$

# Looking from above