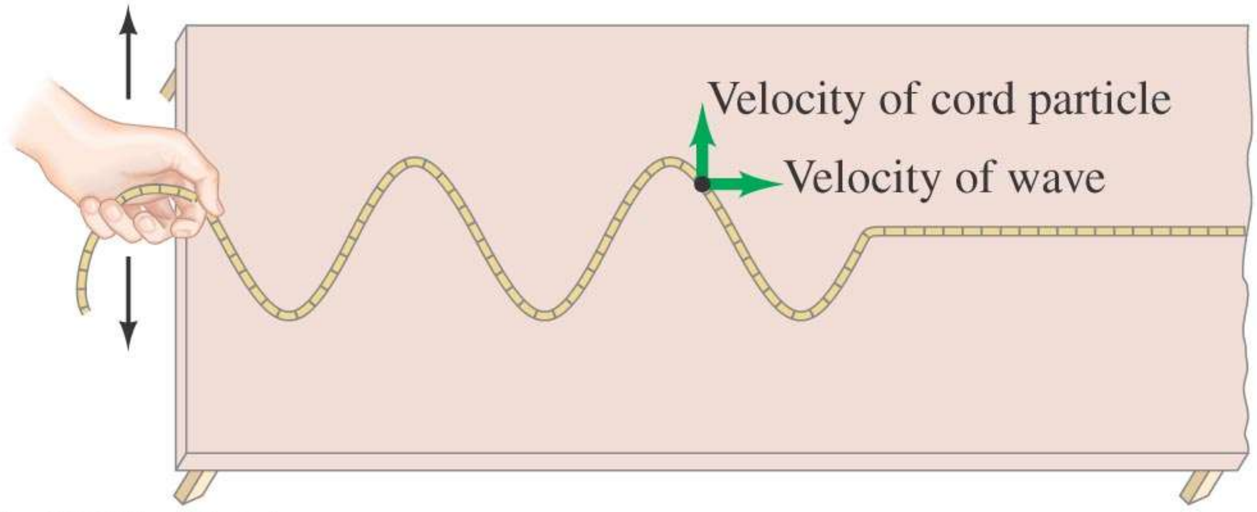
Waves



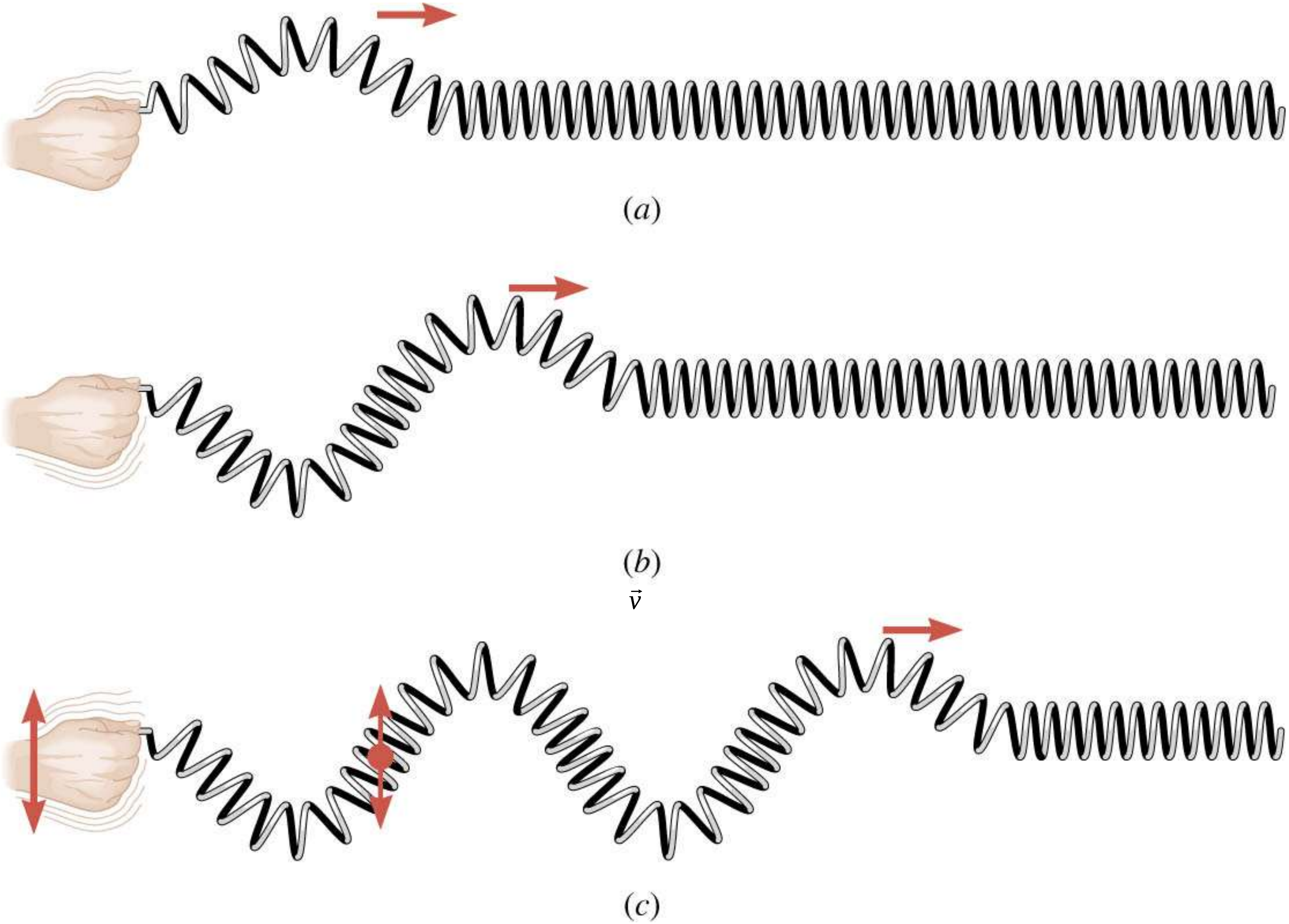
- Average speed of cord particle is zero, just moves up and down.
- We are interested in wave velocity
- Waves carry energy, momentum, but not mass

- Oscillator (person or machine) controls A and f
- Speed v depends on medium and is independent of A and f

$$v_{string} = \sqrt{\frac{Tension}{mass/length}}$$

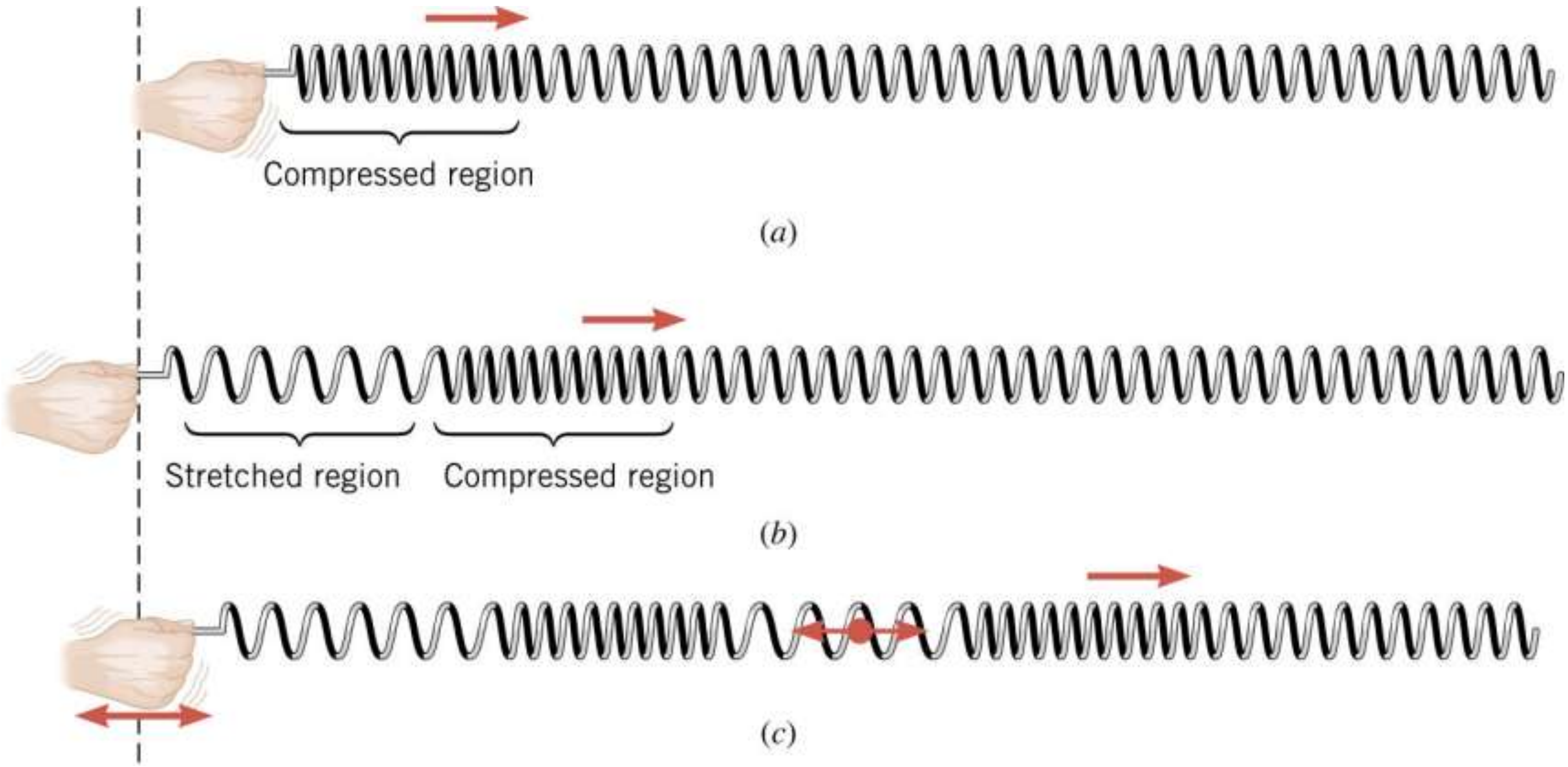
$\mu = \dfrac{mass}{length}$ linear density

v_{air} = 340 m/s but varies with temperature and pressure.



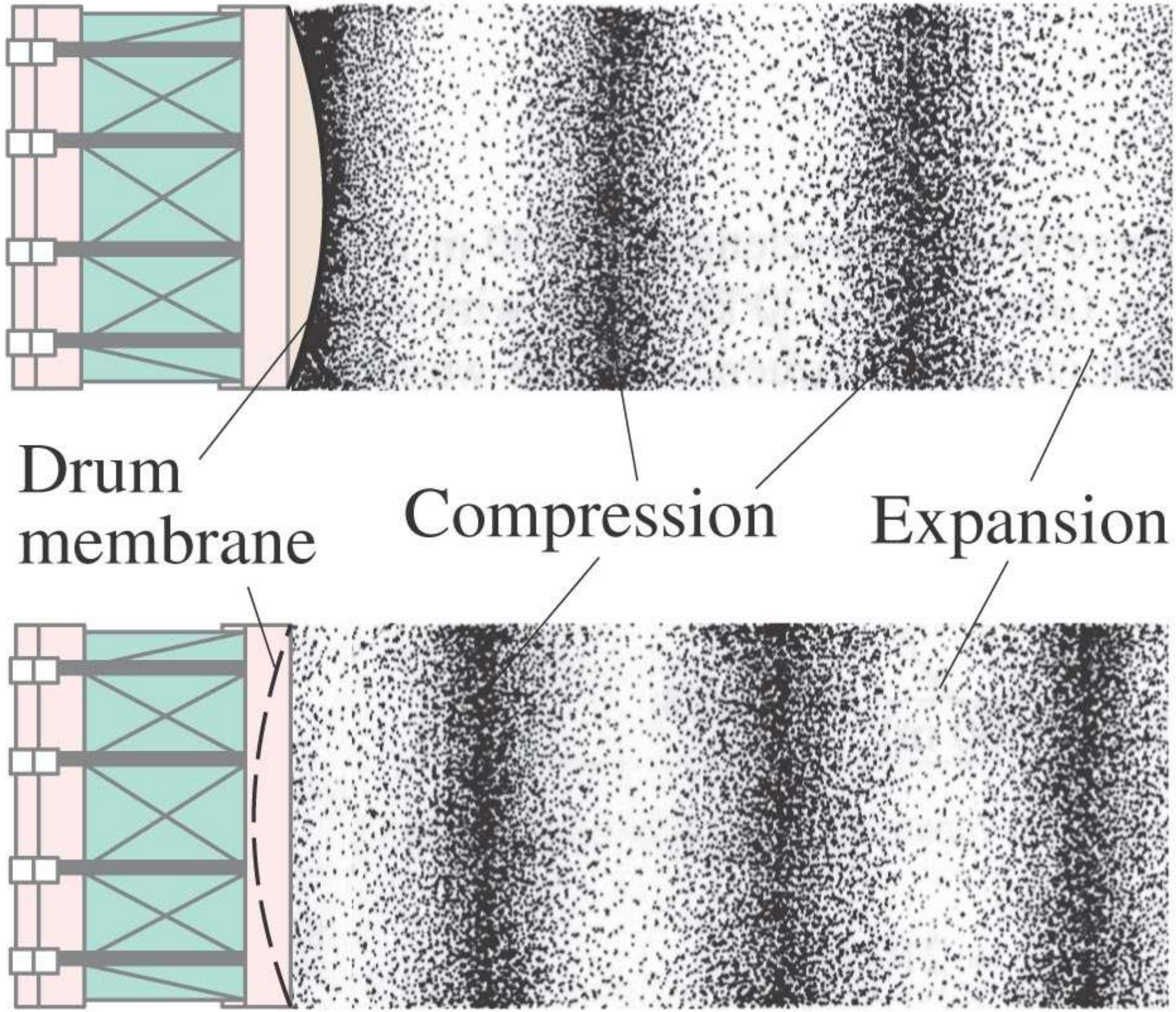
Transverse Wave

Disturbance $\perp \vec{v}$

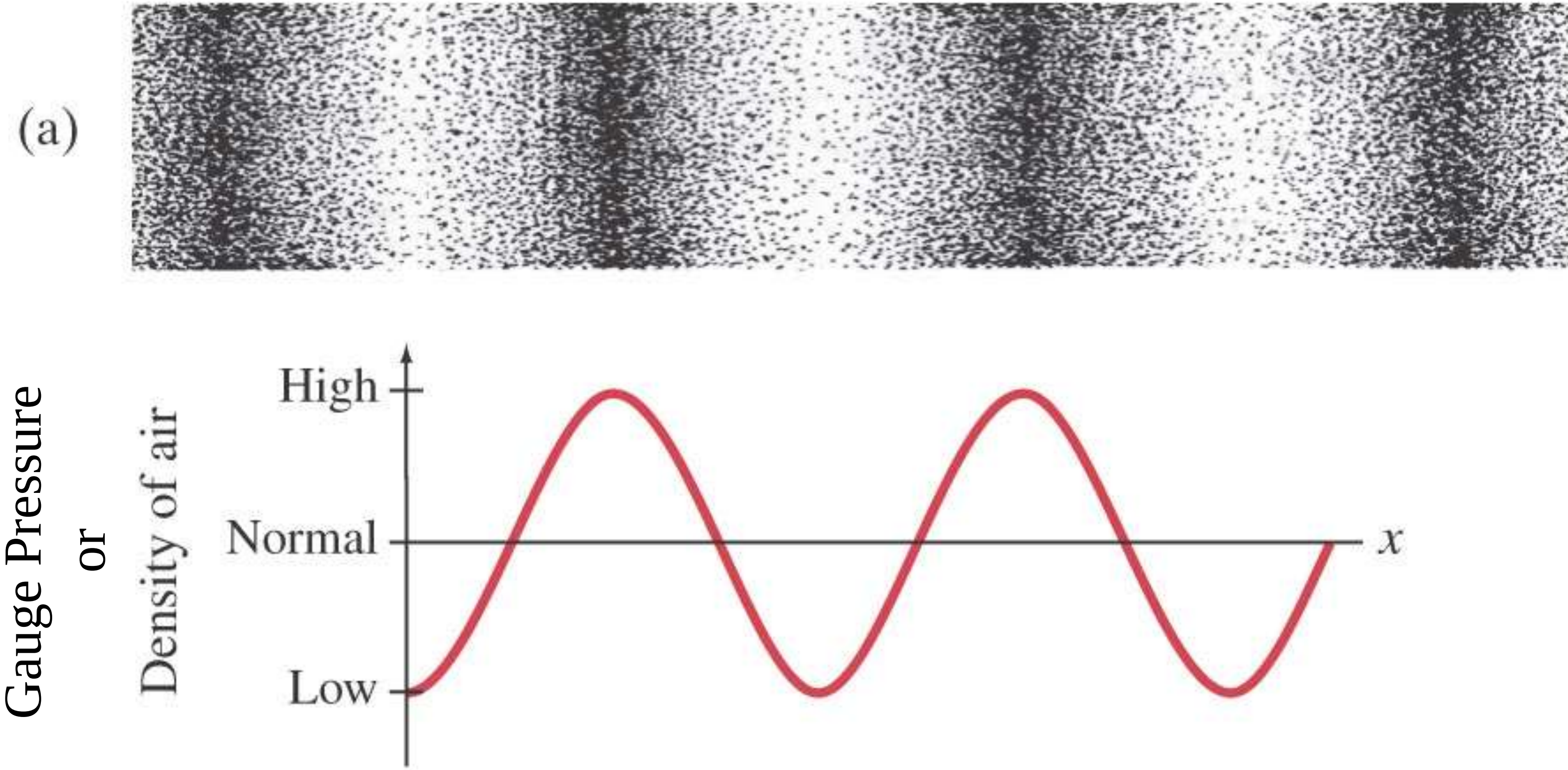


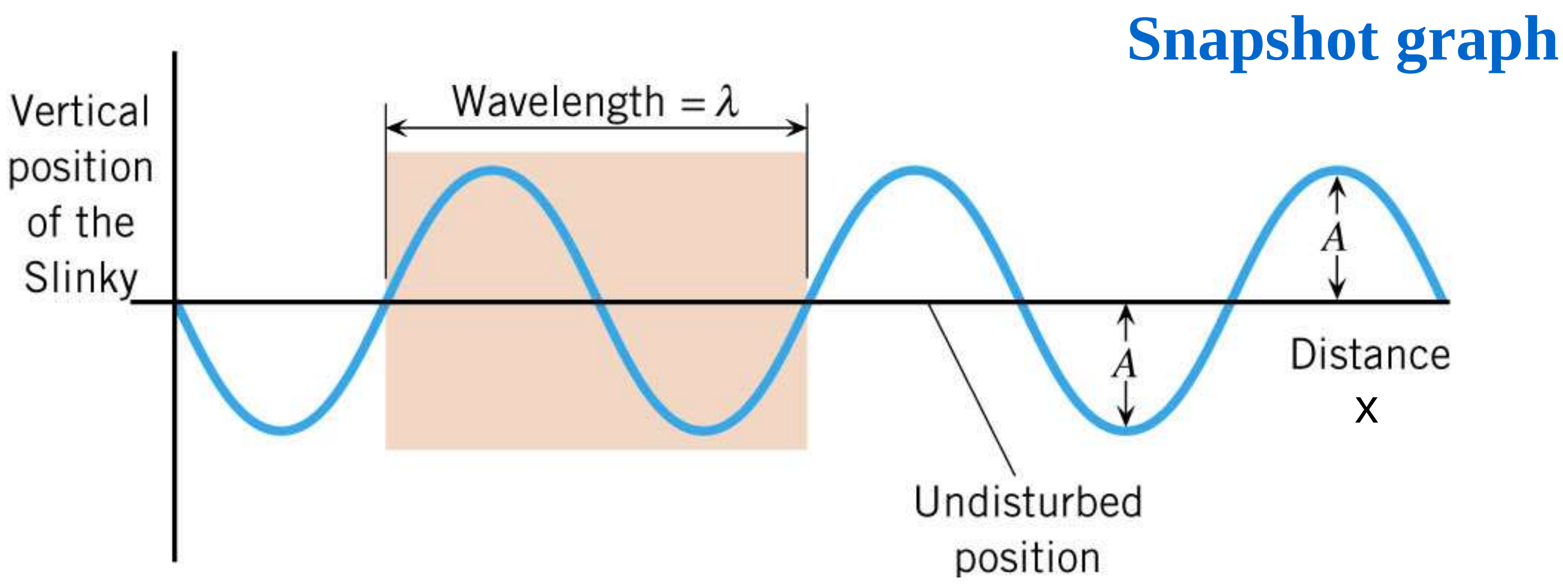
Longitudinal Wave

Disturbance || $\vec{v}$

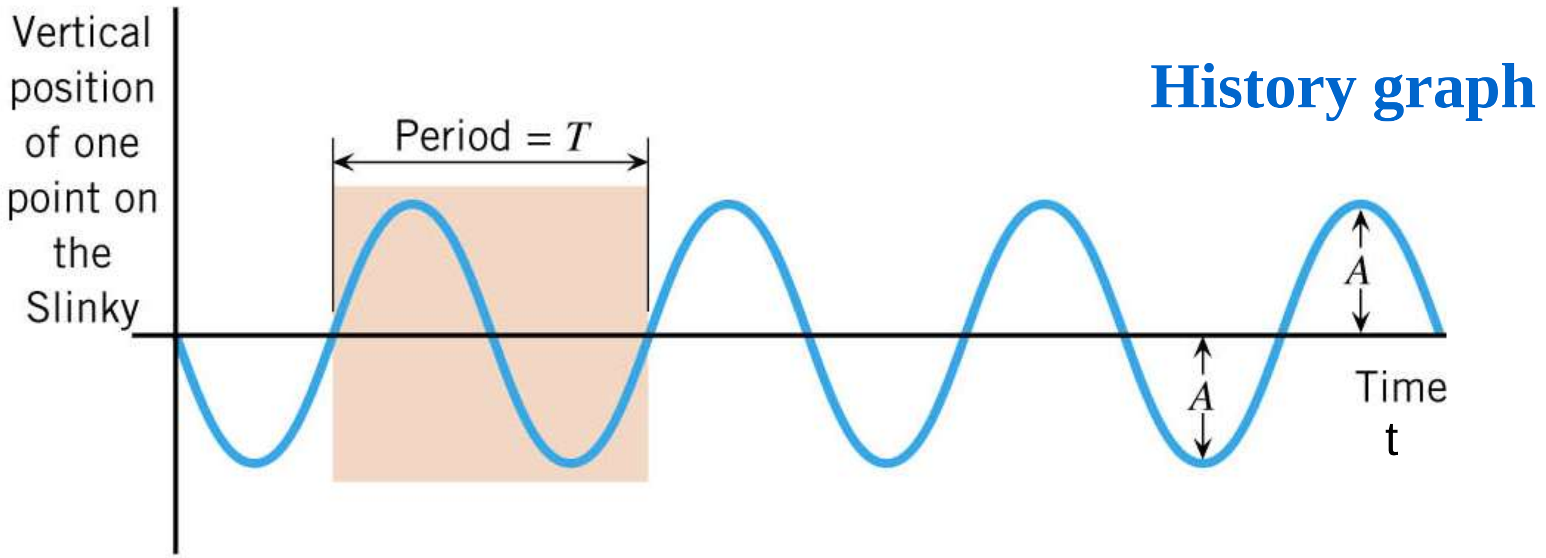

Drum
membrane
Compression
Expansion

(a)

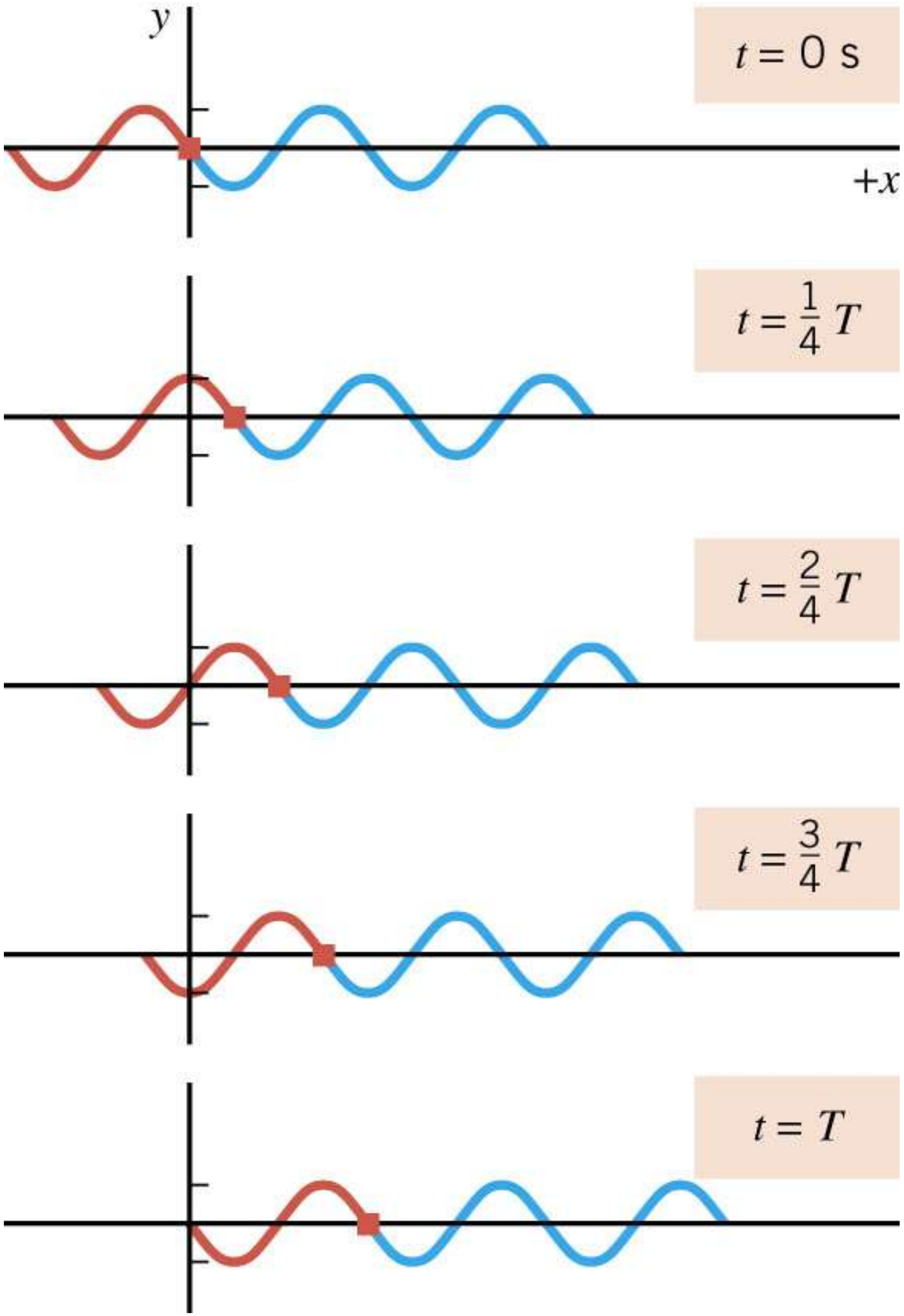




At a particular time

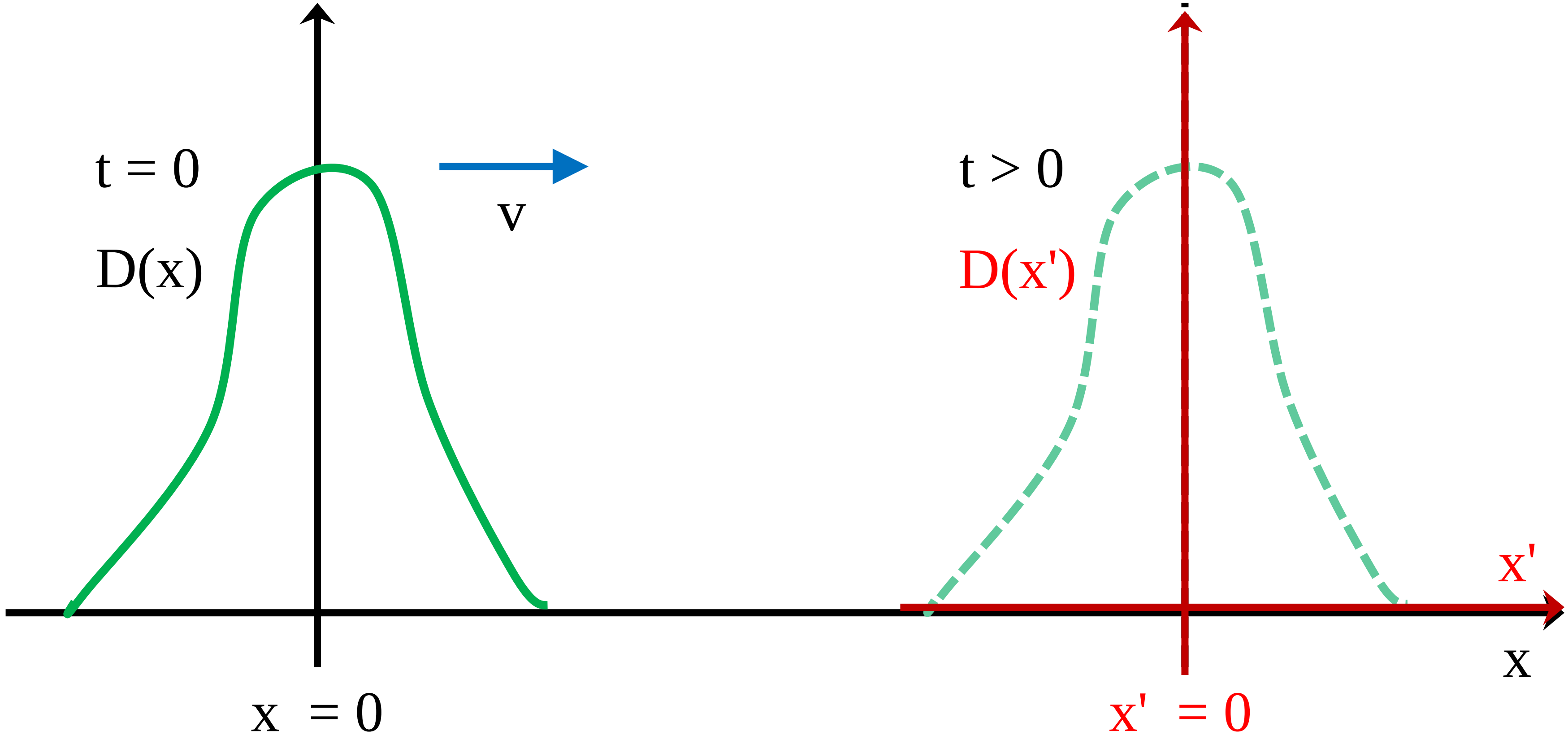


At a particular location



In one period T, wave moves one wavelength λ

$$v = \lambda/T = \lambda f$$



$$D(x') = A\sin(2\pi x'/\lambda)$$

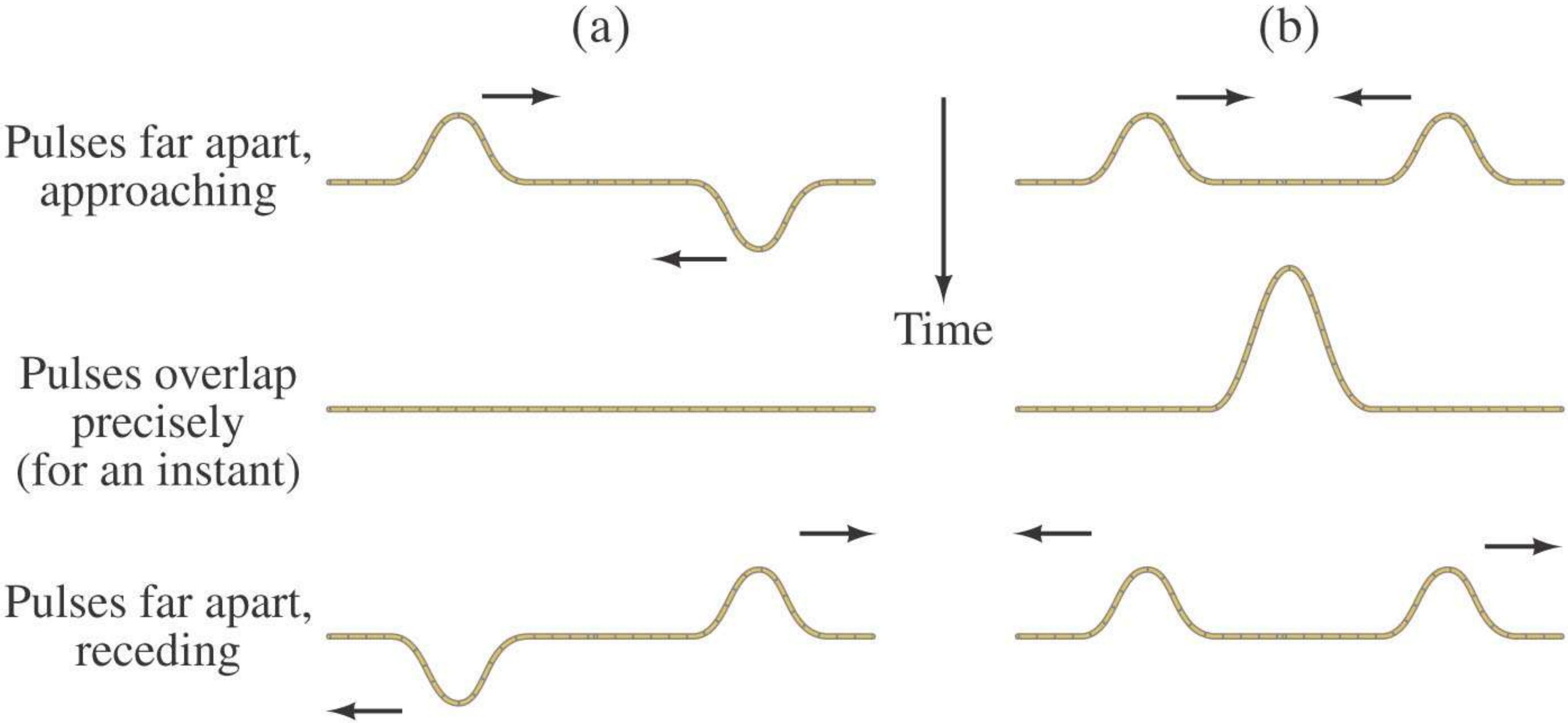
But $x' = x - vt$ which means $D(x')$ is $D(x, t)$!

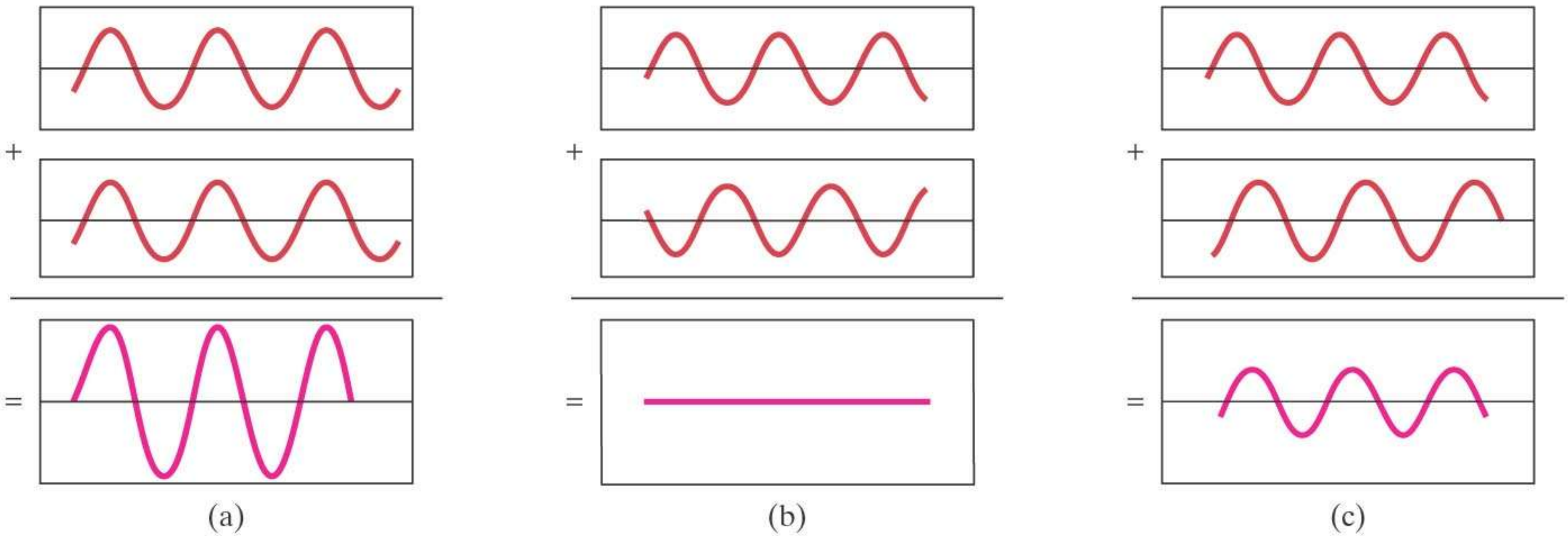
$$D(x, t) = A\sin(\ (2\pi/\lambda)(x - vt)\ )$$

$$D(x, t) = A\sin(\ 2\pi x/\lambda - 2\pi vt\ /\lambda\ )$$

$$D(x, t) = A\sin(\ 2\pi x/\lambda - 2\pi ft\ )$$

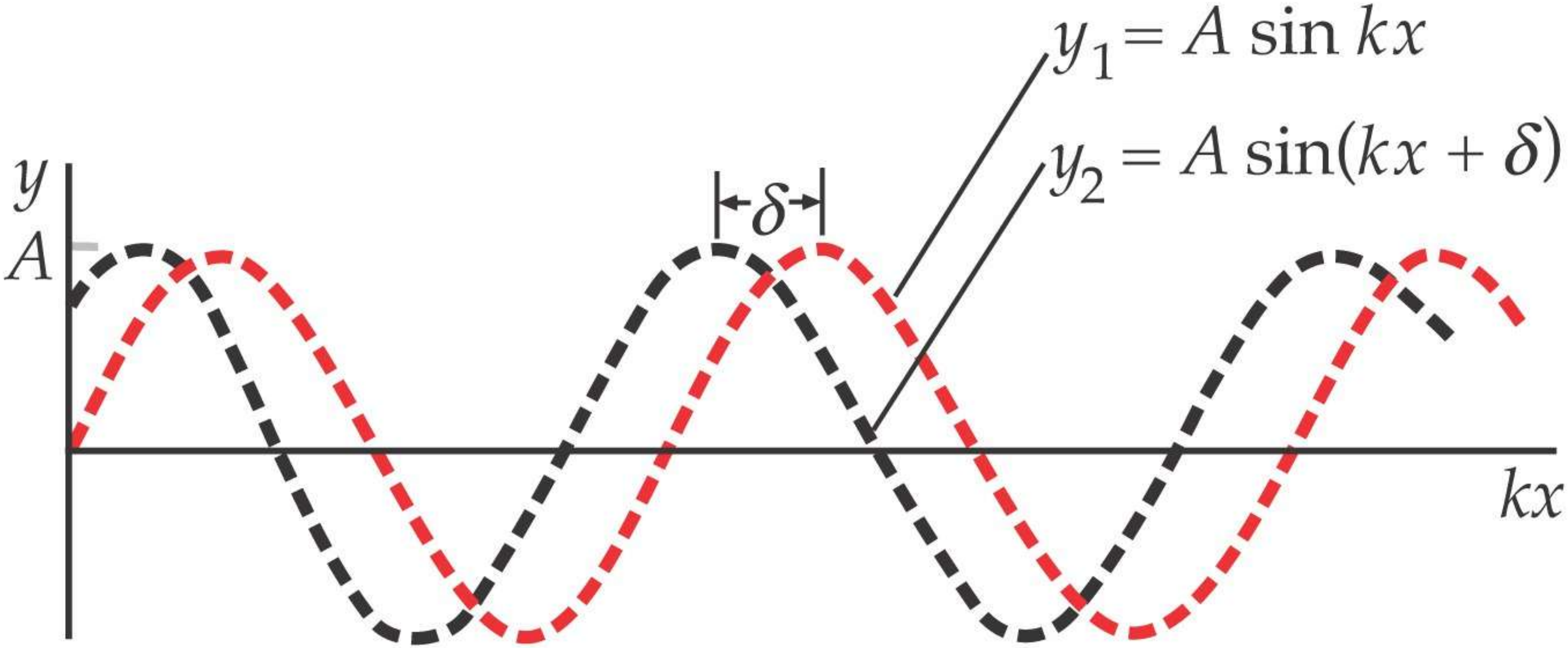
Superposition





Constructive Interference

Destructive Interference

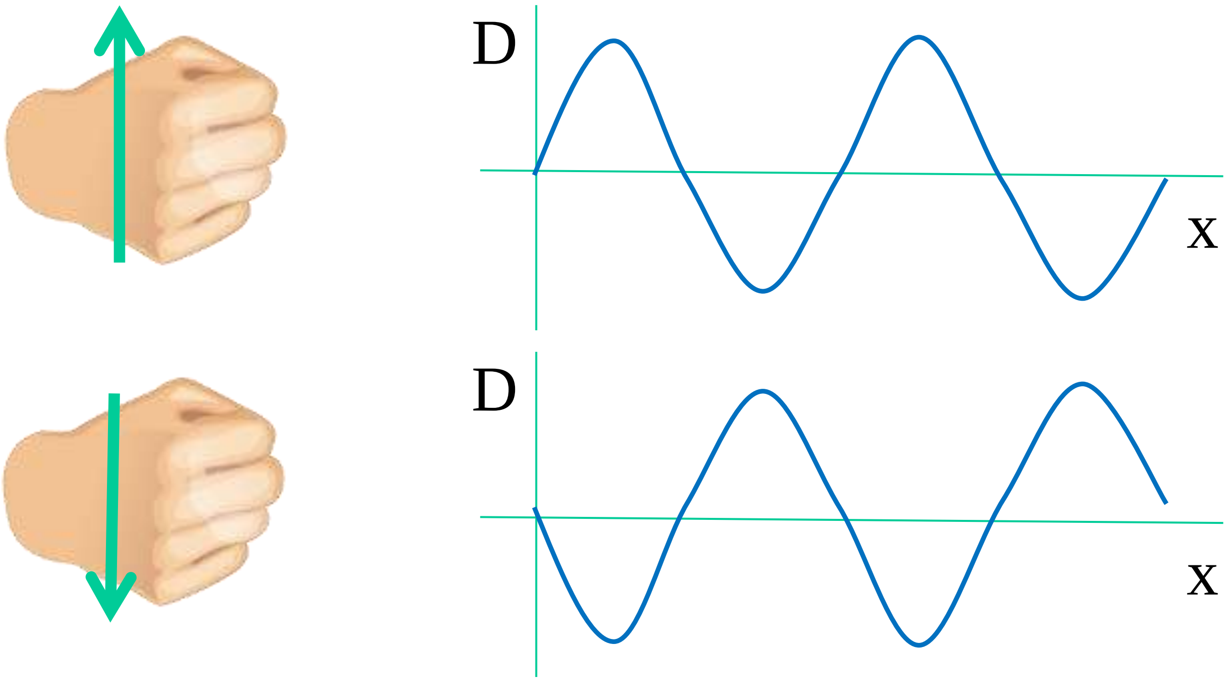


δ = phase difference in radians

= $2\pi\Delta x/\lambda$

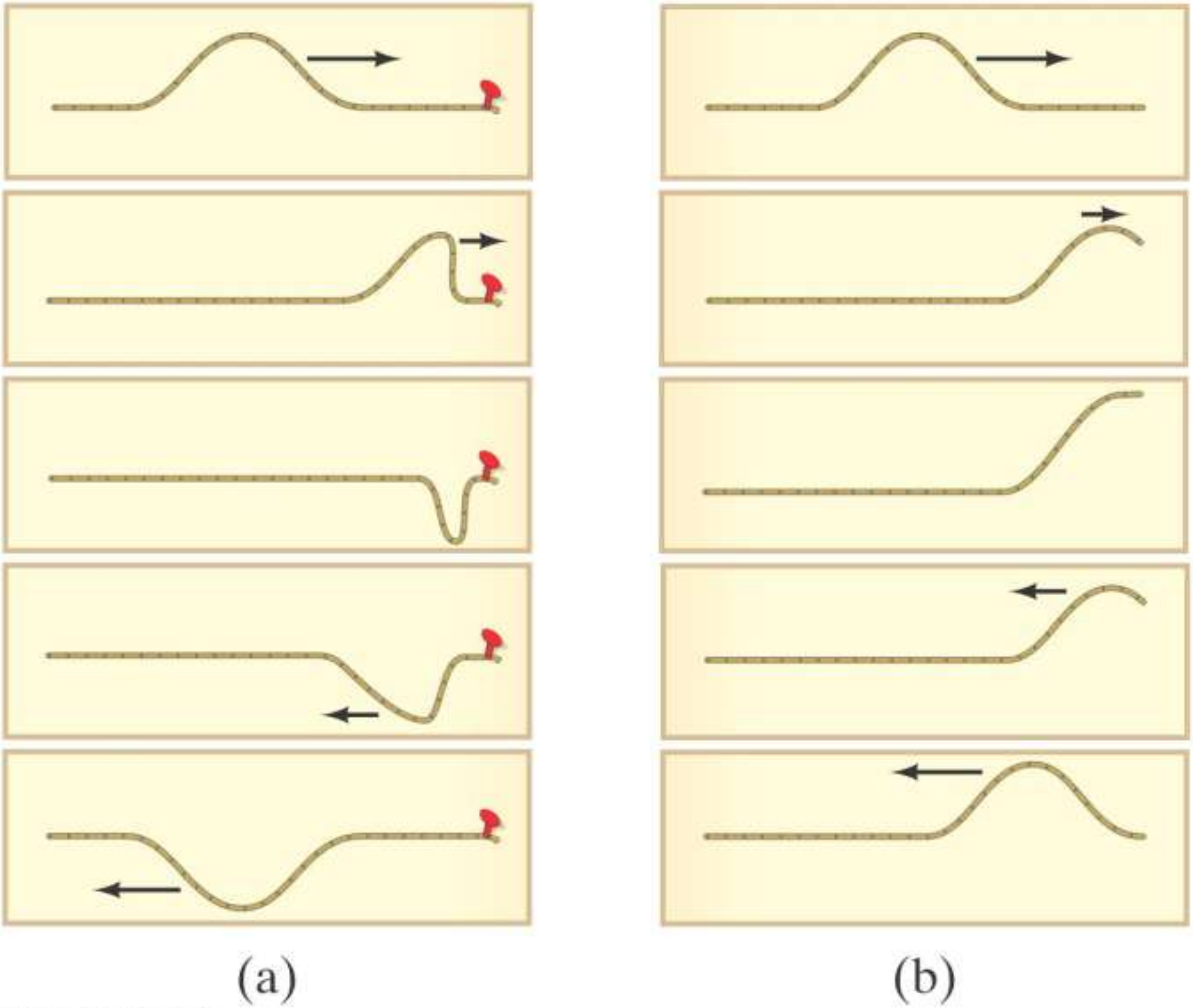
Questions

Initial Phase



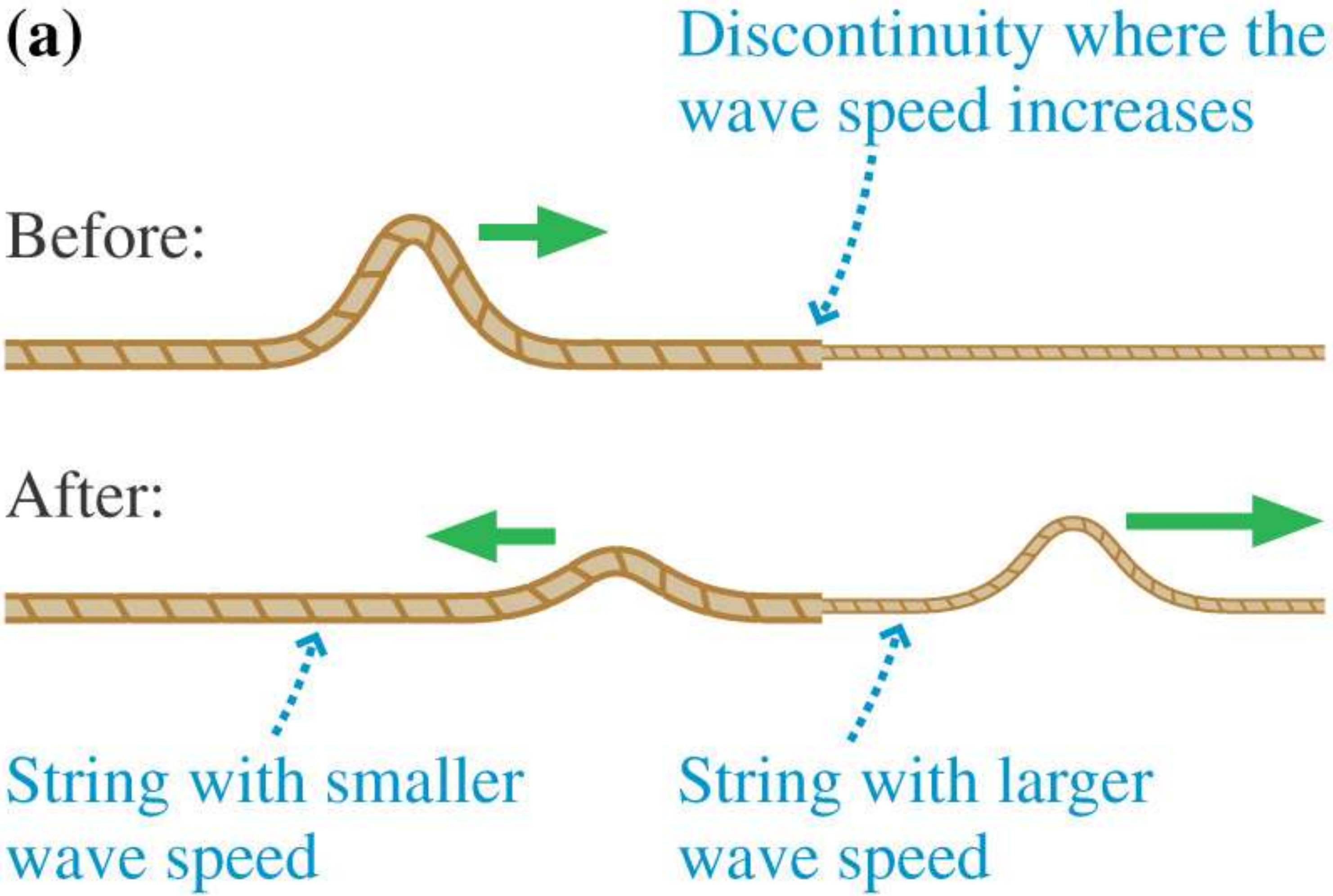
$$\delta = 2\pi\Delta x / \lambda = 2\pi (\tfrac{1}{2}\lambda) / \lambda = \pi$$

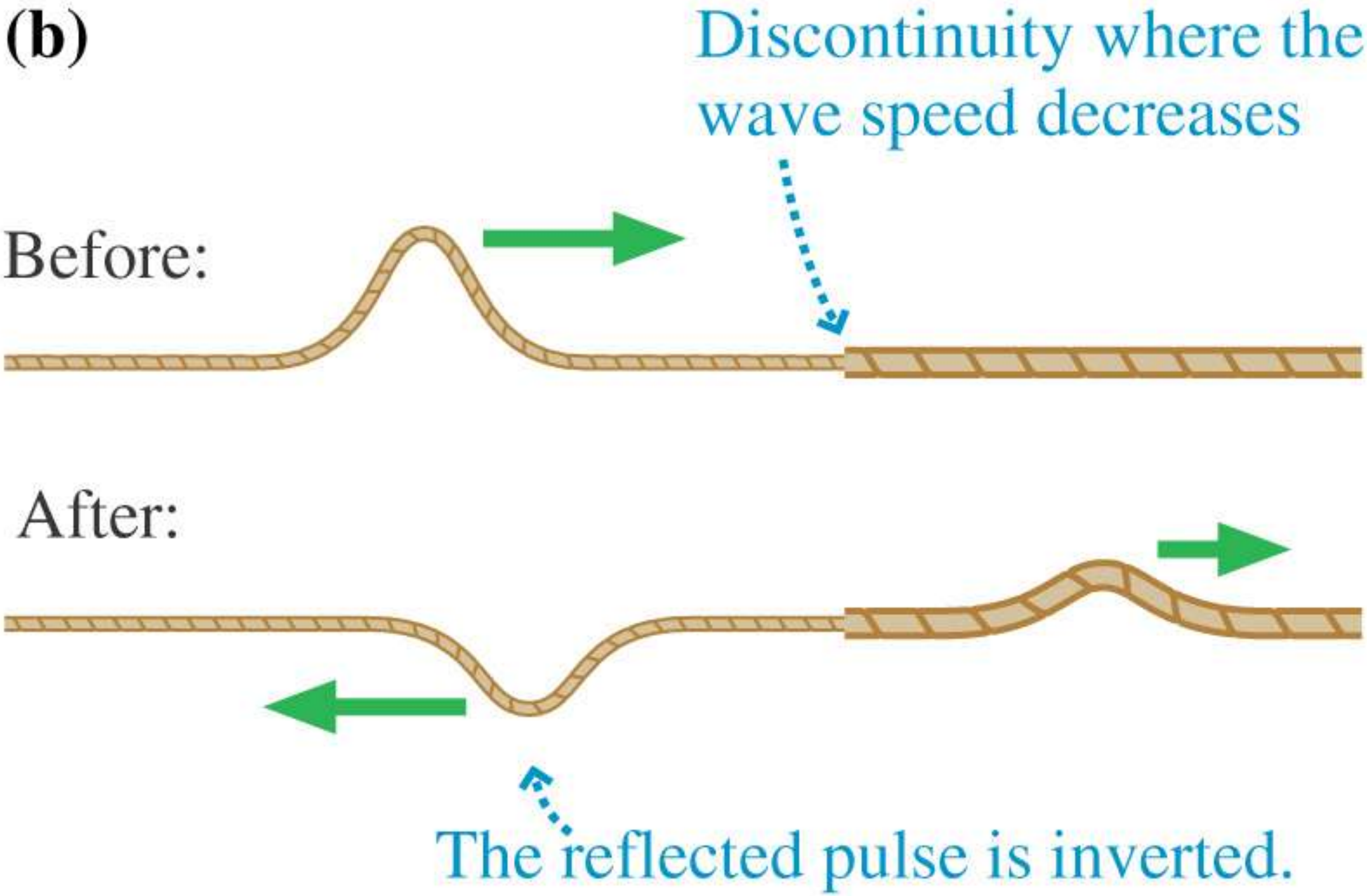
Reflection



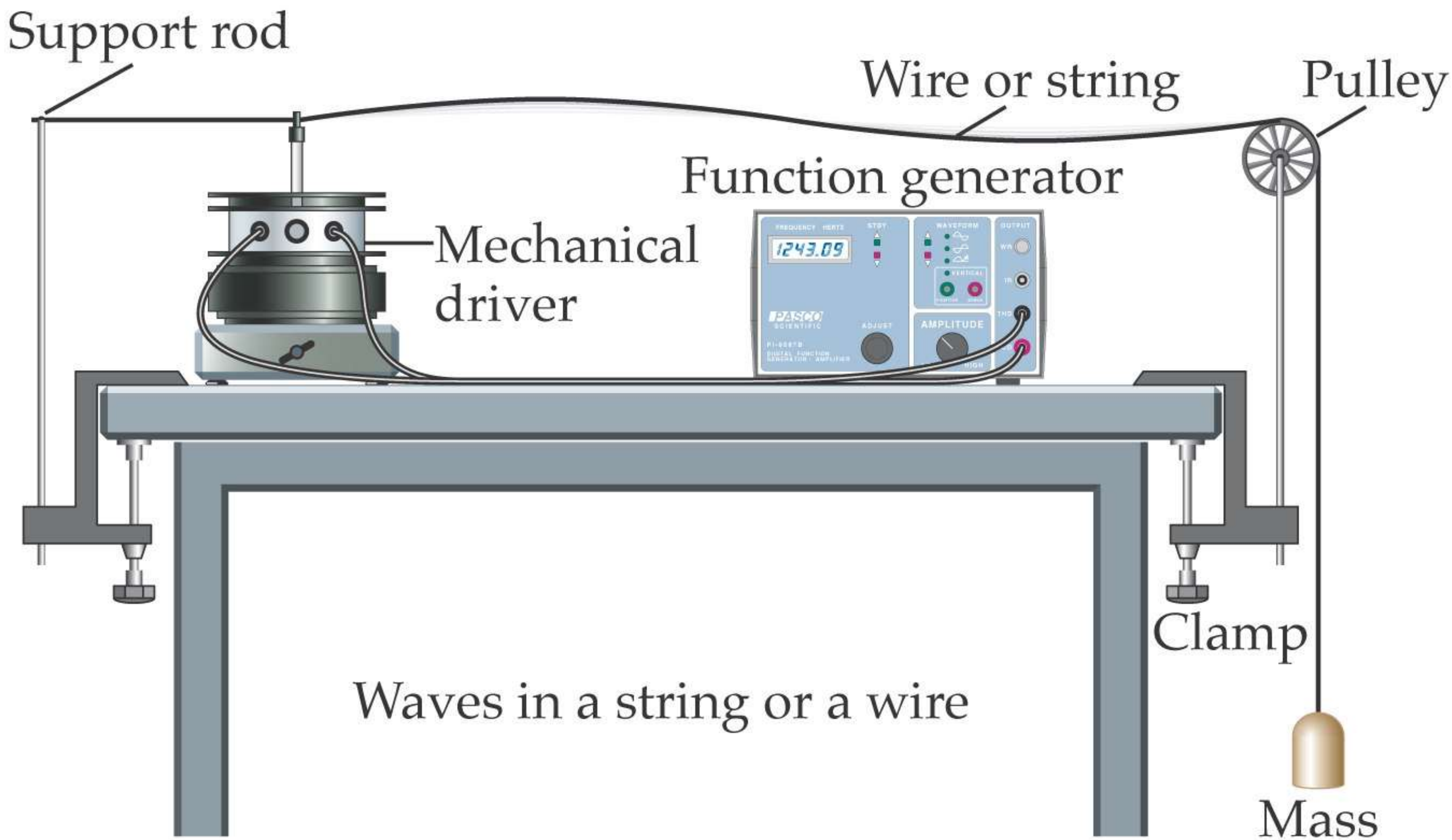
Fixed end

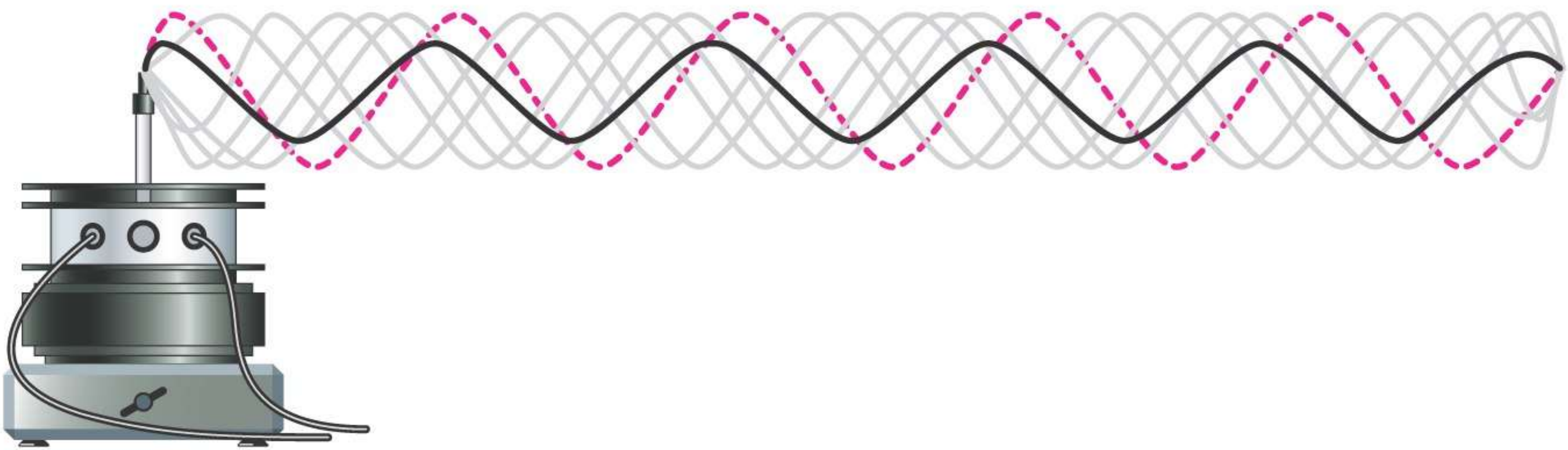
Free end

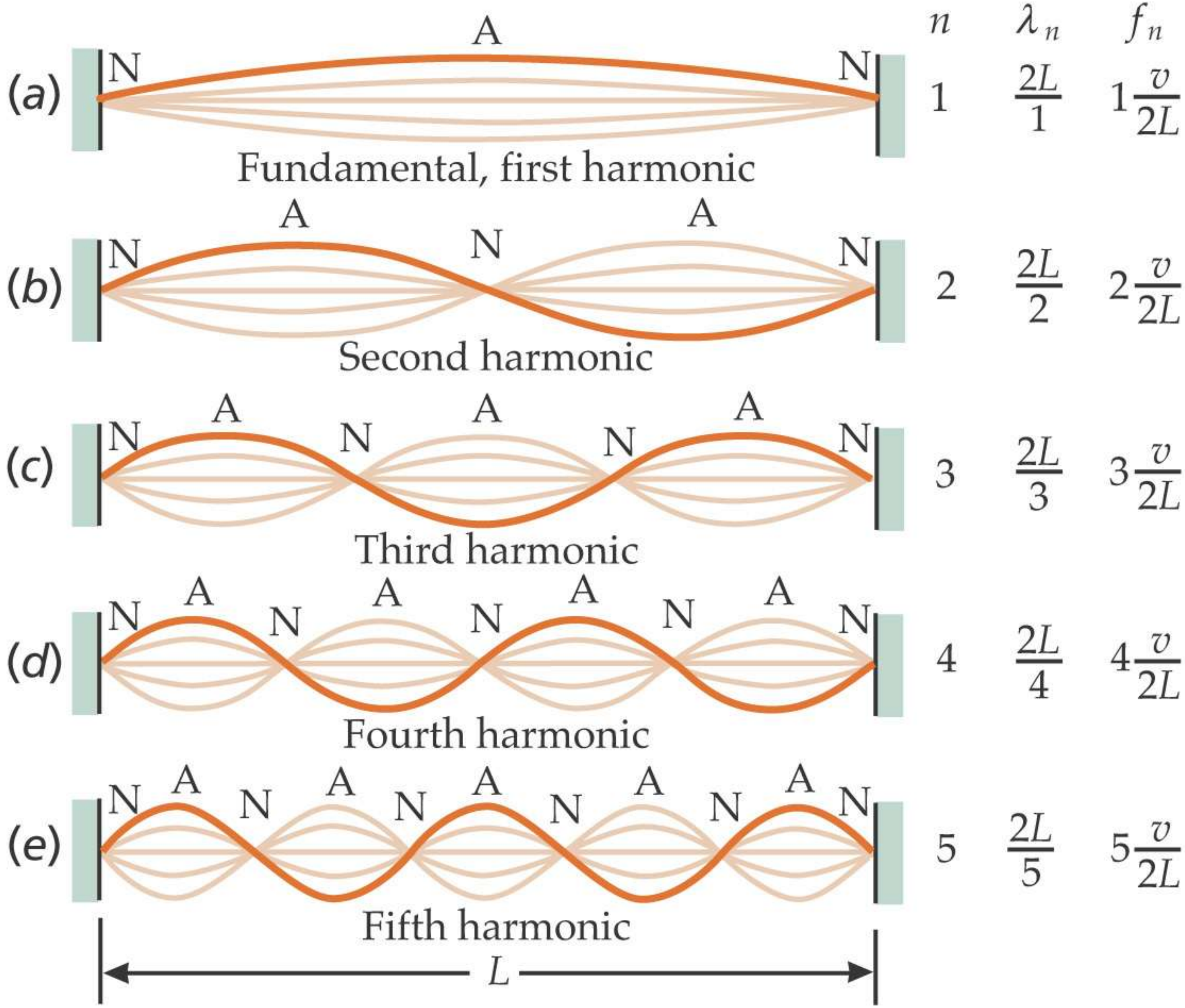

(a)
Discontinuity where the wave speed increases
Before:
After:
String with smaller wave speed
String with larger wave speed


(b)
Discontinuity where the wave speed decreases
Before:
After:
The reflected pulse is inverted.

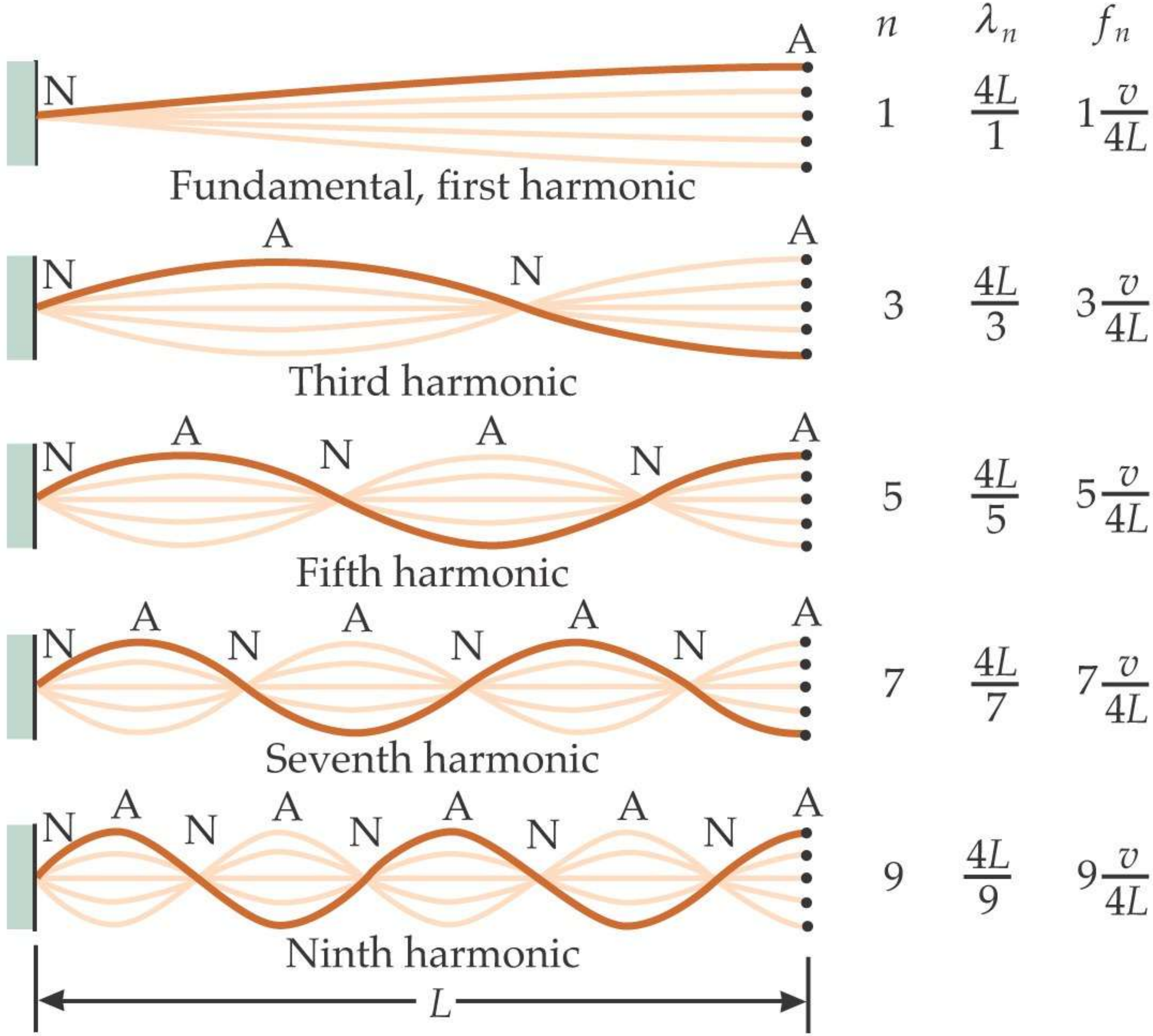
Standing Waves


Support rod
Wire or string
Pulley
Function generator
Mechanical
driver
1243.09
PASCO
AMPLITUDE
Clamp
Mass
Waves in a string or a wire





n
λ_n
f_n
(a)
N
A
N
1
$\frac{2L}{1}$
$1\frac{v}{2L}$
Fundamental, first harmonic
(b)
N
A
N
A
N
2
$\frac{2L}{2}$
$2\frac{v}{2L}$
Second harmonic
(c)
N
A
N
A
N
A
N
3
$\frac{2L}{3}$
$3\frac{v}{2L}$
Third harmonic
(d)
N
A
N
A
N
A
N
A
N
4
$\frac{2L}{4}$
$4\frac{v}{2L}$
Fourth harmonic
(e)
N
A
N
A
N
A
N
A
N
A
N
5
$\frac{2L}{5}$
$5\frac{v}{2L}$
Fifth harmonic
L


n
λ_n
f_n
N
A
1
4L/1
1 v/4L
Fundamental, first harmonic
3
4L/3
3 v/4L
Third harmonic
5
4L/5
5 v/4L
Fifth harmonic
7
4L/7
7 v/4L
Seventh harmonic
9
4L/9
9 v/4L
Ninth harmonic
L

Equation of Standing Wave

Consider two waves travelling in opposite directions

$$D_1(x,t) = A\sin(kx - \omega t)$$

$$D_2(x,t) = A\sin(kx + \omega t)$$

$$D_{Net}(x,t) = ?$$

Recall trig identity

$$sin\theta_1 + sin\theta_2 = 2\cos\frac{\theta_1+\theta_2}{2}\, sin\frac{\theta_1-\theta_2}{2}$$

$$\Rightarrow D_{net}(x,t) = 2A\cos(\omega t)\sin(kx)$$

For many reflections:

$$D_{net}(x,t) = \underbrace{A_{net}\cos(\omega t)}_{\text{Amplitude oscillates}}\underbrace{\sin(kx)}_{\text{Stationary in space}}$$

String fixed at $x = 0$ and $x = L$

At $x = 0$, $\sin(k*0) = \sin(0) = 0$ √

At $x = L$, want $\sin(kL) = 0$ (Node)

Requires $kL = n\pi$, n an integer

$$(2\pi/\lambda)L = n\pi \; \& \; v = \lambda f$$

$$f = nv/\,2L$$

String fixed at $x = 0$ and open at $x = L$

At $x = 0$, $\sin(k*0) = \sin(0) = 0$ √

At $x = L$, want $\sin(kL) = 1$ (Antinode)

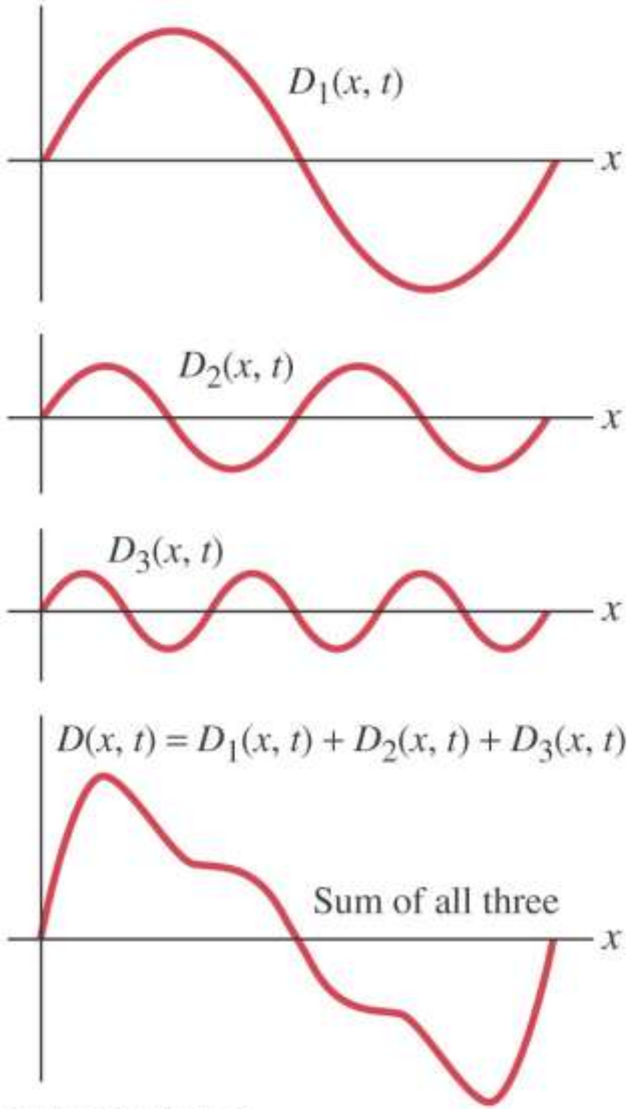
Requires $kL = m\pi/2$, m an odd integer

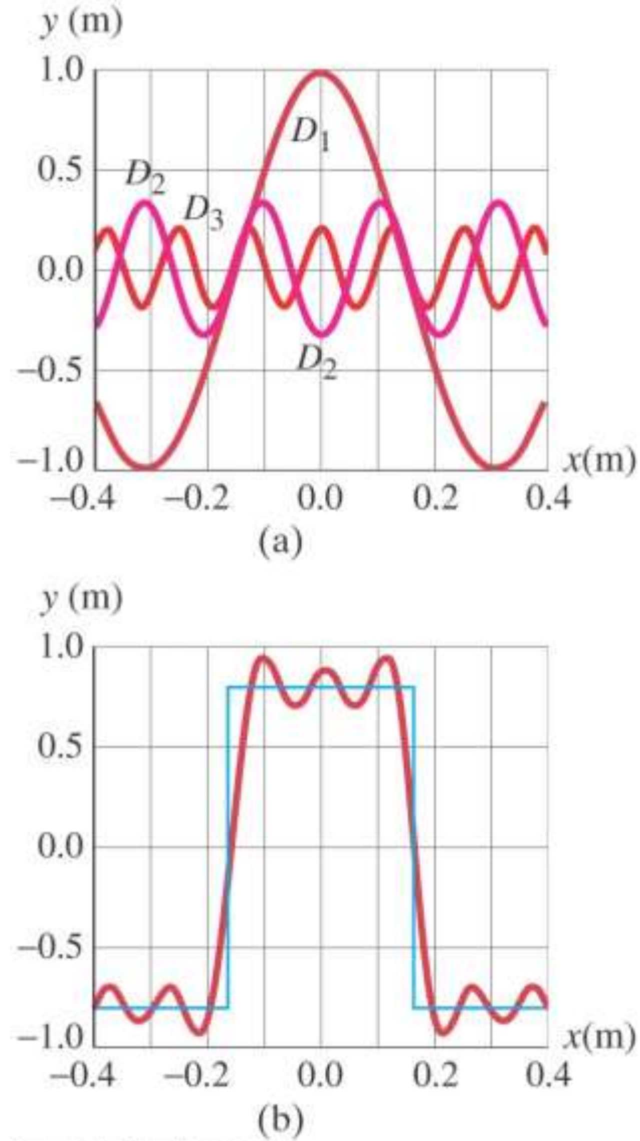
$$(2\pi/\lambda)L = m\pi/2 \ \&\ v = \lambda f$$

$$f = mv/4L$$

Fourier Analysis

- Multiple waves of same frequency on a string can only give sinusoidal shapes
- Waves of different frequencies can give much more complicated shapes
- A Piano and Guitar playing the same note sounds different because of extra harmonics

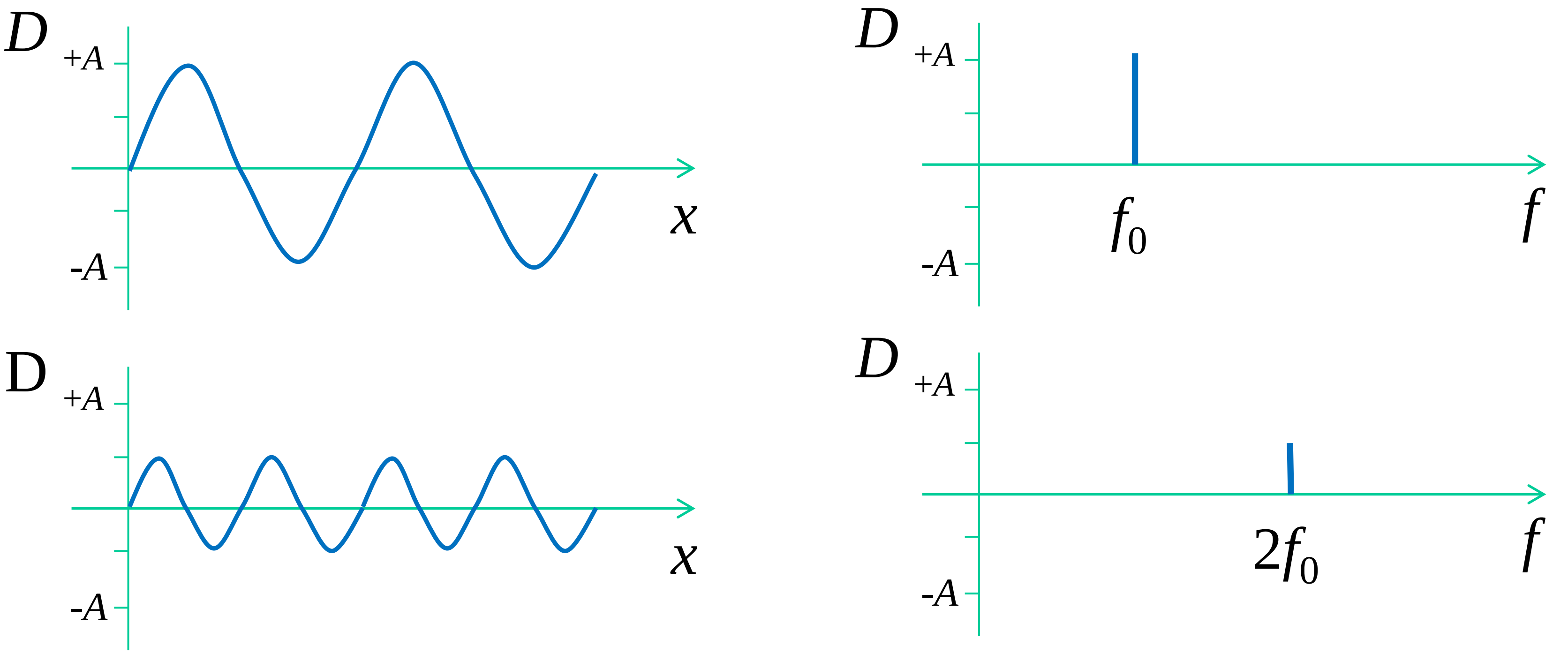

$D_1(x, t)$
x
$D_2(x, t)$
x
$D_3(x, t)$
x
$D(x, t) = D_1(x, t) + D_2(x, t) + D_3(x, t)$
Sum of all three
x


y (m)
1.0
0.5
0.0
−0.5
−1.0
D_1
D_2
D_3
D_2
x(m)
−0.4
−0.2
0.0
0.2
0.4
(a)
y (m)
1.0
0.5
0.0
−0.5
−1.0
x(m)
−0.4
−0.2
0.0
0.2
0.4
(b)

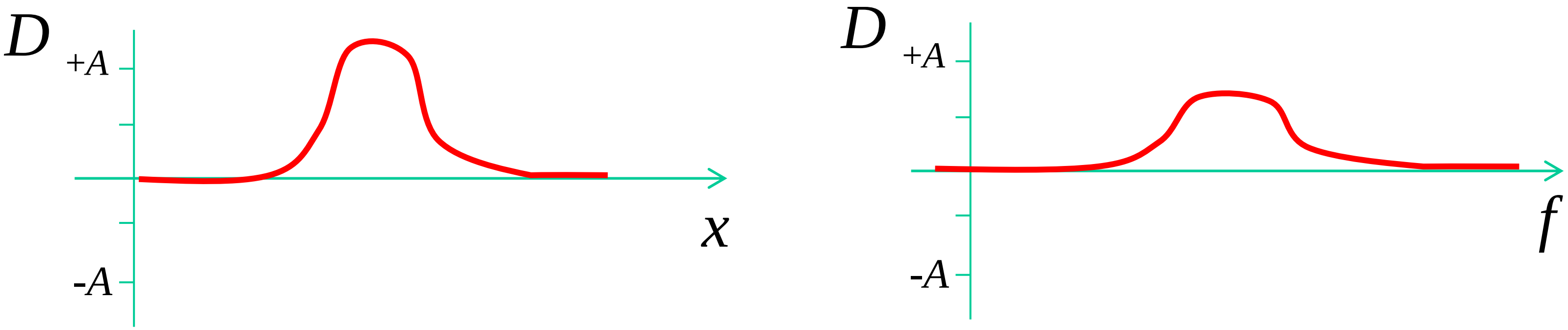
Frequency Representation

Waves on the same string

- Must have same speed *v*
- Differ only in amplitude *A* and frequency *f*

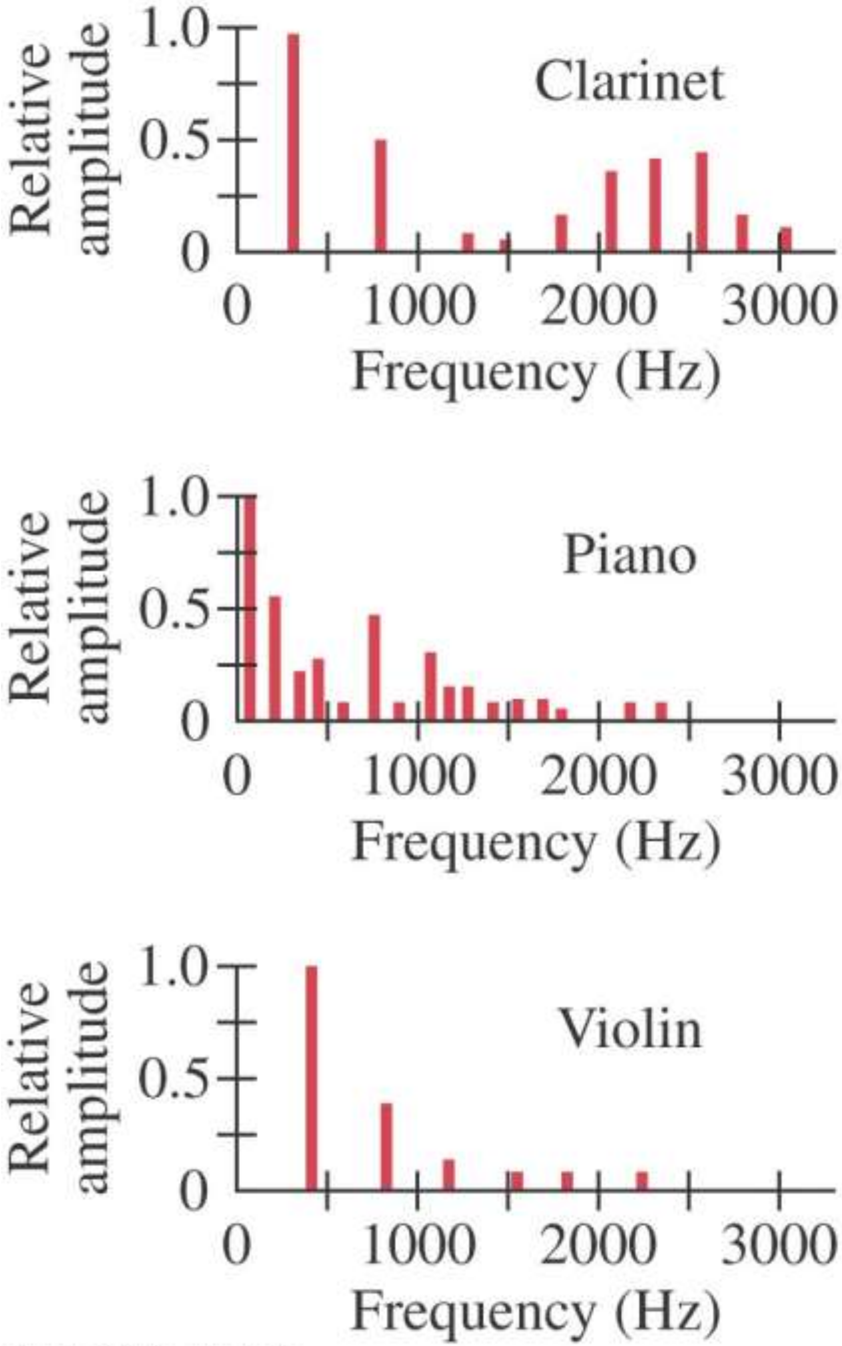


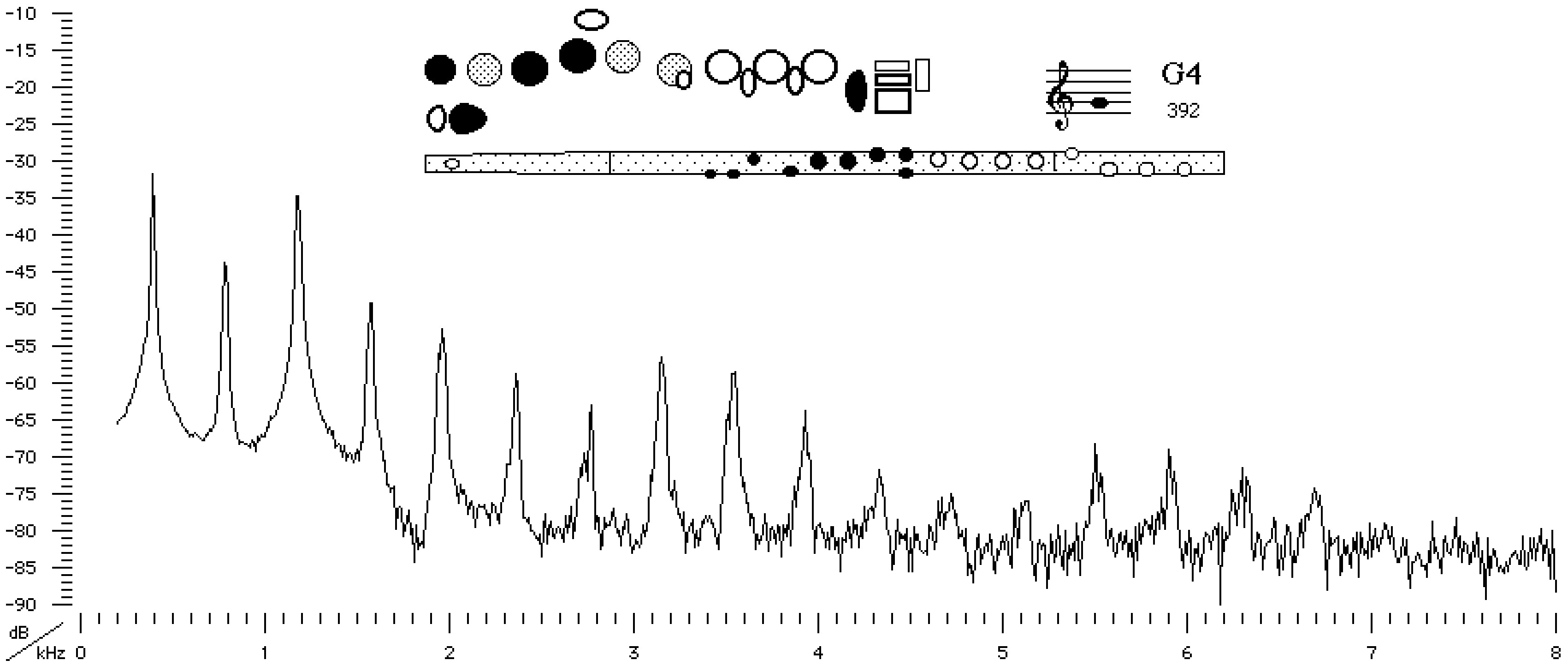
Pulse – Frequency Representation



- Pulses generated by a quick motion
- Would require a range of frequencies to duplicate using F.A.

- Pluck a guitar string, hit a piano key, blow into a tuba
- You generate a pulse (many frequencies)
- Only the frequencies that match the standing wave (or resonant) frequencies of the instrument are persist for a long time before dying out.
- Matched frequencies with can have different amplitudes
- Why different instruments sound fifferent when playing the same note.


Clarinet
Piano
Violin
Relative amplitude
1.0
0.5
0
0
1000
2000
3000
Frequency (Hz)



http://www.phys.unsw.edu.au/jw/graphics/G4.B.sound.gif