Identifying the Major Source of Uncertainty

In the lab you have seen how to estimate the uncertainty in each measurement you make. Typically your measurements are of distance, time, force, mass, etc. You have also learned how to carry the uncertainties in your measurements through your calculations using *uncertainty propagation* and in analyzing linear graphs using limiting lines. From your analysis (calculations and graphs) you indirectly determined some other quantity. The precision (number of significant figures) of this quantity is limited by the precision of your measurements. In particular, the measurement that most limits the precision of your derived quantity, is called the *Major Source of Uncertainty*. It is useful to know because, if you want to improve the precision of your result, this is the measurement that must be made more precise.

Example 1

Consider Newton's Second Law

$$F = ma\,. \qquad [1]$$

We wish to determine the mass m of a cart by measuring the acceleration a for applied force F. (Of course there is an easier way to determine the mass - the weigh scale. But we may wish to reserve that measurement as a comparison to our experimental result. In many cases there is no second method of determining a physical quantity. In that case we must rely on the indirect experimental measurement for everything we know about that quantity.) For a typical experiment we might find that for applied force $F = 2.0 \pm 0.2$ N yields an acceleration of 0.153 ± 0.005 m/s^2. Which of our measurement is the major source of uncertainty?

First we rewrite equation (1) in so that the derived quantity m is written in terms of the measured quantities, F and a.

$$m = F/a\,. \qquad [2]$$

Using uncertainty propagation

$$\begin{aligned}\delta m &= \delta\left[\frac{F}{a}\right] \\ &= \left[\frac{F}{a}\right]\left[\frac{\delta F}{F} + \frac{\delta a}{a}\right] \qquad [3] \\ &= m\left[\frac{\delta F}{F} + \frac{\delta a}{a}\right]\end{aligned}$$

Next we rewrite this equation to get an expression for the relative uncertainty in m in terms of the relative uncertainties in the measured quantities.

$$\frac{\delta m}{m} = \left[\frac{\delta F}{F} + \frac{\delta a}{a}\right] \qquad [4]$$

Next we calculate the relative uncertainties in F and a. These are

$$\delta F/F = 0.2/2.0 \times 100\% = 10\%$$

and

$$\delta a/a = 0.002/0.153 \times 100\% = 1.3\%.$$

Hence the relative uncertainty is m is $\delta m/m = 11\%$. Note that even if we improved the measurement in a by a factor of ten so that $\delta a/a = 0.1\%$, the uncertainty in m is $\delta m/m$ is still about 10%. That is not much of an improvement. Clearly the measurement of F is the major source of uncertainty and we would first need to improve the precision of this measurement.

The second useful thing about identifying the major source of uncertainty is that it suggests how to improve the precision of the quantity. First note that $\delta F/F$ is a ratio. If we want it to be smaller we either (a) make δF smaller or (b) make F bigger. Choice (a) normally means to use more precise instruments which are usually costlier. Choice (b) may be more practical. We simply apply a bigger force to the object. For example, F = 20.0 ± 0.2 N will give an acceleration of 1.532 ± 0.005 m/s^2 and we calculate that

$$\frac{\delta m}{m} = \left[\frac{\delta F}{F} + \frac{\delta a}{a}\right] = \frac{0.2}{20.0} + \frac{0.005}{1.532} = 0.013\,,$$

so that the uncertainty in the mass is now only 1.3%.

Note that major source of uncertainty only applies to quantities that you have directly measured. Suppose that the mass in the example was a cart with a jammed wheel. Now the force that you are pulling the cart with is no longer the net force F required in Newton's Second Law. Therefore, despite the fact that that you measure your applied force precisely, and get a "mass" with plenty of significant figures, your result will be inaccurate because of the systematic uncertainty of the jammed wheel. So when someone asks for the major source of uncertainty, they are asking for the major source of uncertainty in the quantities that you have directly measured. Results that do not agree within experimental uncertainty may be caused by such systematic uncertainties as long as you have made reasonable assumptions about the uncertainty in your measurements.

Example 2

Consider the formula for the period of a pendulum

$$T^2 = \frac{4\pi^2}{g} L. \qquad [5]$$

We measure $T = 0.406\ s \pm 1\%$ and $L = 0.100 \pm 0.001\ m$ and wish to find g.

We rewrite equation [5] to find g

$$g = 4\pi^2 \frac{L}{T^2}. \qquad [6]$$

Using uncertainty propagation we find

$$\frac{\delta g}{g} = \left\{\frac{\delta L}{L} + 2\frac{\delta T}{T}\right\}. \qquad [7]$$

Now are relative uncertainties are $\delta T/T = 1\%$ and

$$\delta L/L = 0.001/0.100 \times 100\% = 1\%\ .$$

Since the relative uncertainties in T and L are the same at 1%, we might be tempted to say that there is no one major source of uncertainty. However, in equation [7] the relative uncertainty in T is doubled, so the measurement of T is the major source of uncertainty.

Note that uncertainty in T is fixed so we would need to acquire a better timer to get more precise results.

Example 3

The surface area of a rectangular block of wood is given by the formula

$$A = 2(LW + LT + WT)\ . \qquad [8]$$

Where we find the length, width, and thickness to be $L = 12.2 \pm 0.1\ cm$, $W = 6.5 \pm 0.1\ cm$, and $T = 0.9 \pm 0.1\ cm$, respectively. The equation for the relative uncertainty in A is

$$\frac{\delta A}{A} = 2\left\{\frac{L(W+T)}{A}\frac{\delta L}{L} + \frac{W(L+T)}{A}\frac{\delta W}{W} + \frac{T(L+W)}{A}\frac{\delta T}{T}\right\}. \qquad [9]$$

Using the data we have been given we find

$$\frac{\delta A}{A} = 2\{0.385 + 0.681 + 0.973\}. \qquad [10]$$

It appears that the thickness is the major source of uncertainty.

Note that since we are measuring the area of a particular block, we have no way of changing the thickness of the block. If we want more precise results in this case, we need a more precise measuring instrument.

Steps to identifying a major source of uncertainty

1. Identify all the directly measured quantities in the experiment.
   - A directly measured quantity is one found by using a piece of equipment such as a weigh scale, a ruler, of a computer timing system.
   - An indirectly measured quantity is calculated from directly measured quantities.
   - As an example consider a cart with a flag passing through a photogate. The width of the flag d and the time it takes passing through the photogate t are directly measured quantities. The velocity $v = d/t$ is an indirectly measured quantity.

2. Determine the average percentage uncertainty in the directly measured quantities.

3. For experiments involving only a calculations and not a graph:
   - Write the equation variables in terms of the directly measured quantities.
   - Do uncertainty propagation on the equation.
   - From the uncertainty propagation formula, find the measurement that has the largest effect on the derived quantity's relative uncertainty.

4. For experiments involving a linear graph:
   - Write the y-axis and x-axis variables in terms of the directly measured quantities.
   - Do uncertainty propagation on these variables.
   - Identify which variable, x or y, has the greatest relative uncertainty.
   - Identify the major source of uncertainty for the y-axis variable.
   - Identify the major source of uncertainty for the x-axis variable.
   - The major source of uncertainty for the slope and intercept will be the major source of uncertainty in whichever variable, x or y, has the greatest relative uncertainty.

5. Always check your major source of uncertainty to see if it is possible to improve its precision by altering how the measurement is done.

Exercises

For each of the following determine the major source of uncertainty in derived quantity. Assuming that the stated uncertainties are fixed by the apparatus used, indicate how you would alter the measurement to make the results more precise.

1. The acceleration, a, of a cart on an inclined plane is given by the formula $a = gsin(\theta)$ where g is the acceleration due to gravity and θ is the angle that the incline makes to the horizontal. The angle is $\theta = 15.5 \pm 0.5°$ and $g = 9.81 \pm 0.01$ m/s^2.

2. The value of the magnetic constant μ_0 can be determined from the formula for the magnetic field, B, at the centre of a circular loop of wire of radius R and carrying current I

$$B = \frac{\mu_0 I}{2\pi R}.$$

The current $I = 5.21 \pm 0.02$ A, the radius $R = 0.0150 \pm 0.0001$ m, and $B = (7.01 \pm 0.11) \times 10^{-5}$ T.

3. The acceleration due to gravity g can be determined from the formula for the velocity v of a ball bearing on a banked curve given by $v^2 = Rg\tan(\theta)$,R is the radius of curvature of the banked curve, and θ is the angle at which the curve banks, i.e. makes to the horizontal. The velocity v is measured by a flag of width $d = 0.050 \pm 0.001$ cm passing through a photogate timer. The time is $t = 0.109$ s $\pm$ 1%, $R = 15.2 \pm 0.2$ cm , and $\theta = 15.5 \pm 0.5°$.

4. The acceleration a of a glider on an inclined frictionless airtrack may be determined from the formula $v_f^2 = v_i^2 + 2aD$, where the final and initial velocities are measured by a flag of width $w = 0.050 \pm 0.001$ cm passing through two photogate timers and assuming $v = w/t$ where t is the timer reading. D is the separation of two photogate timers. The initial timer reads $t_1 = 0.150$ s $\pm$ 1% while the second reads $t_2 = 0.098$ s $\pm$ 1%. The photogates have separation $D = 30.2 \pm 0.1$ cm.

5. The value of the magnetic constant μ_0 can be determined from the formula for force of attraction F between two straight wires of length L separated by a distance r and each carrying an equal but opposite current I

$$F = \frac{\mu_0}{2\pi}\frac{I^2 L}{r}.$$

The force F is measured by the tiny amount of weight $W = mg$ needed to balance the attraction. It is found that $m = (0.30 \pm 0.01) \times 10^{-3}$ kg, $g = 9.81 \pm 0.01$ m/s^2, $L = 0.350 \pm 0.005$ cm, $I = 17.2 \pm 0.1$ A, and $r = 0.0075 \pm 0.0001$ m.