Physics Formulas And Linear Graphs

One way we investigate the validity of a formula is to take multiple measurements and plot the results as a straight-line or linear graph. It takes some thought and practice to see that almost any physics formula can be plotted in such a way as to produce a linear graph.

Example 1

Newton's Second Law is given by the formula

$$F = Ma \qquad 1.$$

where F is the net force accelerating the system, a is the acceleration of the system, and M is the mass of the system. In a typical experiment a hanging mass, m_h, is connect to a cart, mass M_{cart}, and hung over a pulley. The weight of the hanging mass, $W = m_h g$, is the net force on the system as it accelerates both its own mass and the cart as shown below in Diagram 1. By moving masses from the cart to the hanger, we measure different accelerations. The mass of the system, $M = m_h + M_{cart}$, is kept constant for all runs.

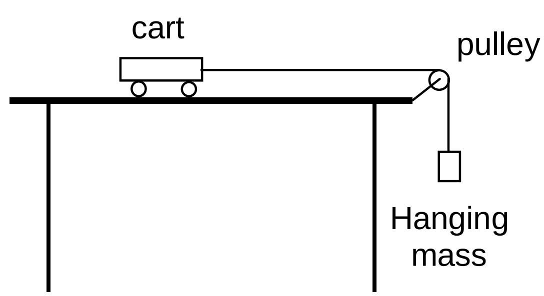

Diagram 1.

There are four questions we need to be able to answer before we can draw the graph and compare our experimental results to theory. The first two questions are:

1. What do we plot on the horizontal or x axis?

2. What do we plot on the vertical or y axis?

When we have the data plotted and a best-fit line drawn to the data, we need to answer two more questions before we can analyze the results. They are:

3. What should the slope of the straight line be?

4. What should the y-intercept of the straight line be?

These four questions are all answered by comparing the physics formula to the equation of a straight line,

$$y = mx + b \qquad \text{2a.}$$

which can also be written as

$$y = b + mx. \qquad \text{2b.}$$

To compare the physics formula with the straight-line equation, write the equation of the line above the formula as shown below

$$\begin{matrix} y & = & m & x & + & b \\ F & = & M & a & & \end{matrix} \qquad \text{3a.}$$

Clearly the force F, the weight of the hanging mass, must be plotted on the y axis.

It is a little harder to determine what should be plotted on the x axis, M or a. But recall we have multiple measurements of the acceleration, a, and only one value of the mass of the system, M. Therefore a is plotted on the x axis.

Having decided what is plotted on the x axis immediately tells us that the slope m of the straight line must equal to the mass of the system, $m = M$.

Deciding what the y-intercept is may be puzzling since $F = Ma$ doesn't seem to have one. However recall that we can add zero to any quantity without changing it, so equation 3a can be written as

$$\begin{matrix} y & = & m & x & + & b \\ F & = & M & a & + & 0 \end{matrix} \qquad \text{3b.}$$

Now it is clear that our experimental y-intercept should be equal to zero, $b = 0$.

Example 2.

A glider is placed on a level, frictionless, airtrack and given a push. After the push ends, there is no net force acting on the glider and it should move with constant velocity according to Newton's First Law, the Law of Inertia. According to kinematics, the position of the glider should be given by the formula

$$x = x_0 + v_0 t \qquad \text{4.}$$

We measure the position x at various times as the glider moves along the track. What are the answers to our four questions in this case?

Again we write the formula and the equation of a straight line.

$$y = m\ x + b$$
$$x = x_0 + v_0 t \qquad \text{5a.}$$

Oops, we have something odd on the right hand side above, the *mx* term should not be over the single quantity x_0. This is a case where we write the equation of the straight line in its alternate form, Eq. 2b.

$$y = b + m\ x$$
$$x = x_0 + v_0\, t \qquad \text{5b.}$$

Now we see that position *x* is plotted the y-axis (confusing isn't it). Since time is measured more than once, it must be plotted on the x axis. That means that the slope of our line must give us the velocity v_0 of the glider. Finally the y-intercept must be the starting position of the glider, $b = x_0$.

Example 3.

In Coulomb's Law, the force *F* between two charges Q_1 and Q_2 separated by a distance *r*, as shown in diagram 2, is given by the formula

$$F = kQ_1Q_2/r^2. \qquad \text{6.}$$

where *k* is the Coulomb constant, $k = 9.11 \times 10^{11}\ N\text{-}m^2/kg^2$. We measure the charge on two balls, Q_1 and Q_2. Then by using a spring scale we measure the force *F* on one charged ball as the distance *r* between the charged balls is changed. What are the answers to our four questions in this case?

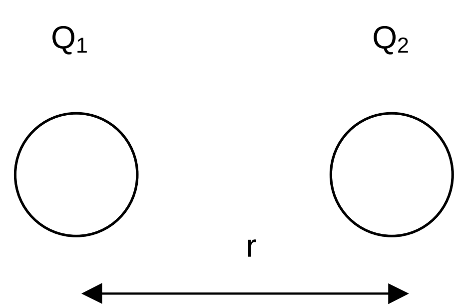

Diagram 2

Again we write the formula and the equation of a straight line.

$$y = m \quad x \quad + \quad b$$
$$F = kQ_1Q_2/r^2 \quad + \quad 0 \qquad \text{7a.}$$

Well now we can see that the force *F* must be plotted on y axis and that the y-intercept of our straight line should be zero, $b = 0$. However, what are m and x? Remember that r is the varied quantity while k, Q_1, and Q_2 are constants in this experiment. We need to isolate the part involving r from k, Q_1, and Q_2 by rewriting the formula as

$$y = m \quad x \quad + \quad b$$
$$F = (kQ_1Q_2)(1/r^2) \quad + \quad 0 \qquad \text{7b.}$$

Now we see that $1/r^2$, not just r, must be plotted on the x axis and the slope of our line will give $m = kQ_1Q_2$.

Actually, in this case, we have a little flexibility in what we plot on the x axis. For instance if we wanted to compare the theoretical value of k with the value we find from experiment, we might prefer to plot Q_1Q_2/r^2 on the x axis. As a result the slope would be exactly k as can be seen from 7c below. That would make comparing experiment to theory slightly easier.

$$y = m \quad x \quad + \quad b$$
$$F = k\,(Q_1Q_2/r^2) \quad + \quad 0 \qquad \text{7c.}$$

Some points to remember:

- You made need to add zero to the formula to find the y-intercept.
- The quantities that are measured multiple times will be end up on the y and x axes
- The measured quantities may be multiplied by constants, squared, or cubed, or inverted, or otherwise manipulated before they are plotted.

Exercises

In the following experiments, the experimental data needs to be converted into a linear graph. In each case, answer the four questions:

1. What do we plot on the horizontal or x axis?
2. What do we plot on the vertical or y axis?

3. What should the slope of the straight line be?
4. What should the y-intercept of the straight line be?

a. The velocity of a glider on an inclined frictionless airtrack is given by the formula

$$v = v_0 + at.$$

The velocity v is measured at various times t. We are interested in measuring the acceleration.

b. The velocity of a glider on an inclined frictionless airtrack is given by the formula

$$v^2 = v_0^2 + 2a\Delta x.$$

The velocity v is measured at various positions x along the airtrack. The displacement is the difference is initial and final positions $\Delta x = x_f - x_i$. We are interested in measuring the acceleration.

c. The magnetic field B at the centre of a circular loop of wire of radius R and carrying current I is given by the formula

$$B = \mu_0 I/2\pi R.$$

where μ_0, the permeability of free space, is given by $\mu_0 = 1.2566 \times 10^{-6}$ $T\text{-}m/A$. While the current I is kept constant, the radius of the circular loop is varied and the magnetic field B is measured. We are interested in measuring μ_0.

d. The magnetic field B at the centre of a circular loop of wire of radius R and carrying current I is given by the formula

$$B = \mu_0 I/2\pi R.$$

where μ_0, the permeability of free space, is given by $\mu_0 = 1.2566 \times 10^{-6}$ $T\text{-}m/A$. While the radius R of the circular loop is kept constant, the current I in the loop is varied and the magnetic field B is measured. We are interested in measuring μ_0.

e. The force of attraction F between two straight wires separated by a distance r and each carrying an equal but opposite current I is given by the formula

$$F = \mu_0 I^2 L/2\pi r^2.$$

where μ_0, the permeability of free space, is given by $\mu_0 = 1.2566 \times 10^{-6}$ $T\text{-}m/A$. For fixed length L of wire and current strength I, the force F between the wires is measured as their separation r is changed. We are interested in measuring μ_0.

f. The acceleration, a, of a cart on an inclined plane is given by the formula

$$a = g\sin\theta$$

where g is the acceleration due to gravity and θ is the angle that the incline makes to the horizontal. As the angle θ is changed, the acceleration a is measured. We are interested in measuring g.

g. The velocity v of a ball bearing on a banked curve is given by the formula

$$v^2 = Rg\tan\theta$$

where g is the acceleration, R is the radius of curvature of the banked curve, and θ is the angle at which the curve banks, i.e. makes to the horizontal. The velocity v is measured as the banking angle θ is changed. The radius of curvature R is kept constant. We are interested in measuring g.

h. The velocity v of a ball bearing on a banked curve is given by the formula

$$v^2 = Rg\tan\theta$$

where g is the acceleration, R is the radius of curvature of the banked curve, and θ is the angle at which the curve banks, i.e. makes to the horizontal. The velocity v is measured as the radius of curvature R is changed. The banking angle θ is kept constant. We are interested in measuring g.