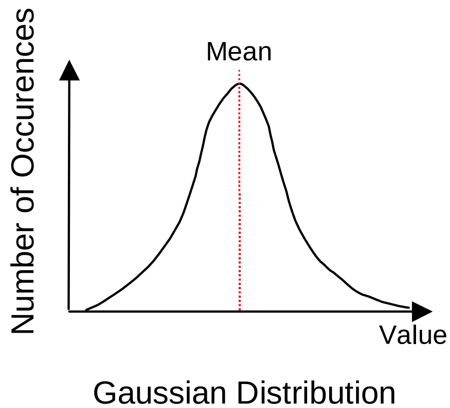
Checking Validity of Uncertainty Assumptions:
Error Bars and Best-Fit Lines

When we make a measurement, we give the range of values A ± ΔA that we expect all subsequent measurements to fall within. The distribution of these measurements might look like Figure 1. If we make enough measurements, statistics tells us that the distribution should be Gaussian, Figure 2. In general, we expect most of our measurements to be near the principle value of our initial measurement with a few lying farther out. However, if our estimate of our uncertainty was correct they should all fall with the range given by our error bars.

Figure 1.

Figure 2.



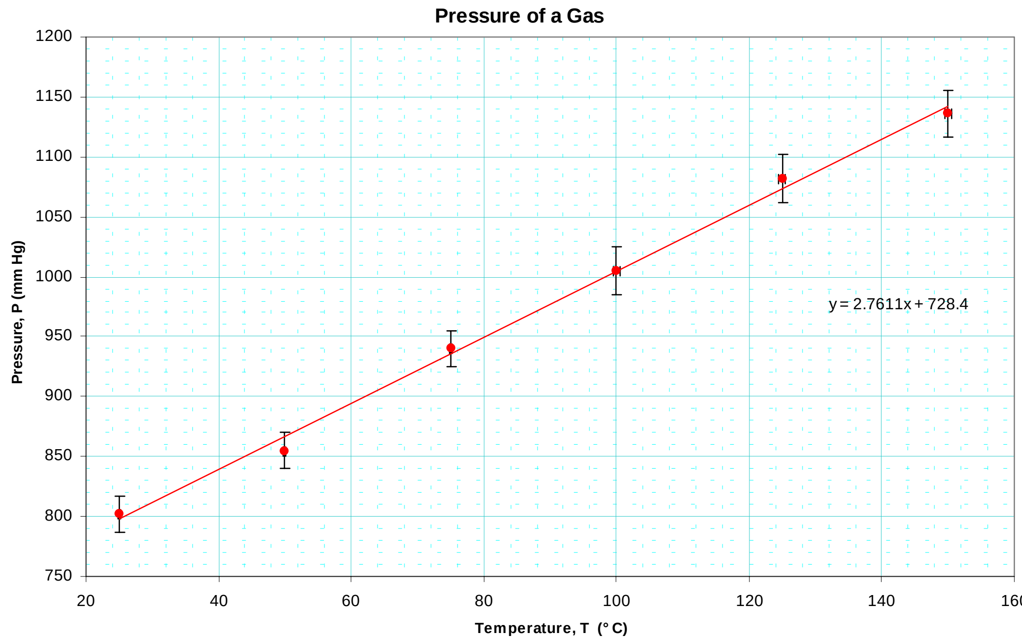
The same distribution should hold when we draw a straight line through a graph of a set of data points. The best-fit line is an average or mean of the data, so the distribution of the data points about the best-fit line should also be Gaussian. This means that if we examine how far each data point is from the best-fit line, its deviation from the best-fit line, we would expect most of the deviations to be small as most data should cluster near the average. However, a few data points would lie farther out.

We can use this behaviour to tell us two things. First, it let's us know how reasonable our uncertainty estimates were. If the deviation is about the same size as the error bars, then our estimate was reasonable. If the deviation is much smaller than our error bars, we have overestimated the error. If the deviation is much larger, then we have either underestimated the size of the uncertainty or have neglected a source of error. Second, as a consequence of the first point, we can also say whether the uncertainty we find in the slope and intercept of the straight line is reasonable, underestimated, or overestimated.

So by looking at our graph we can tell that we have good data and, as a result, can confidently expect our results to agree with accepted values.

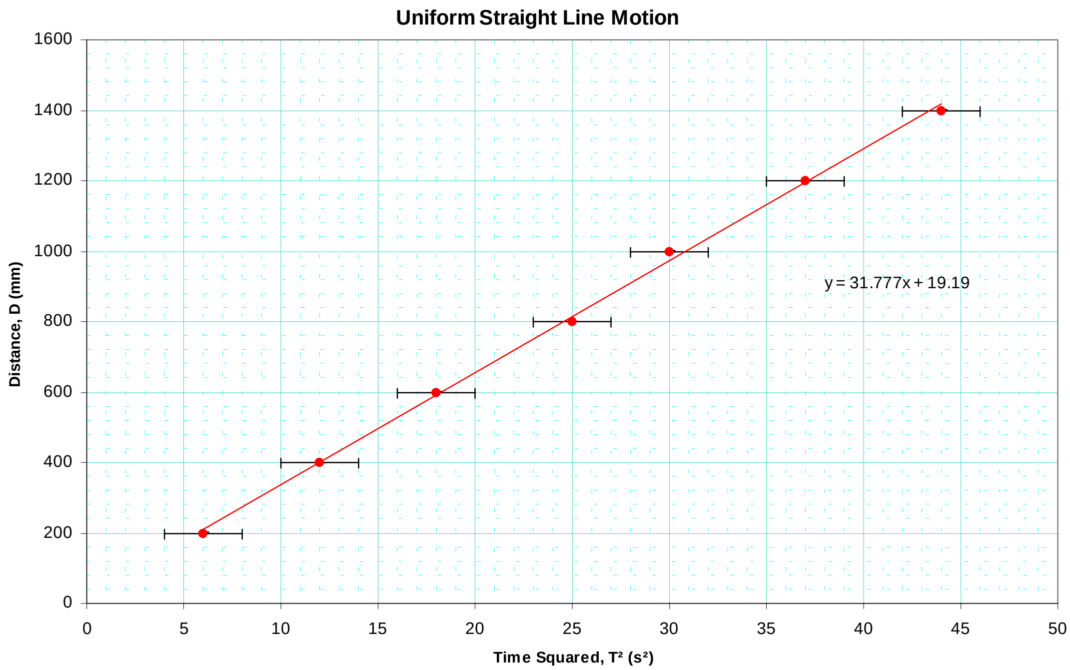
Examples

(a)



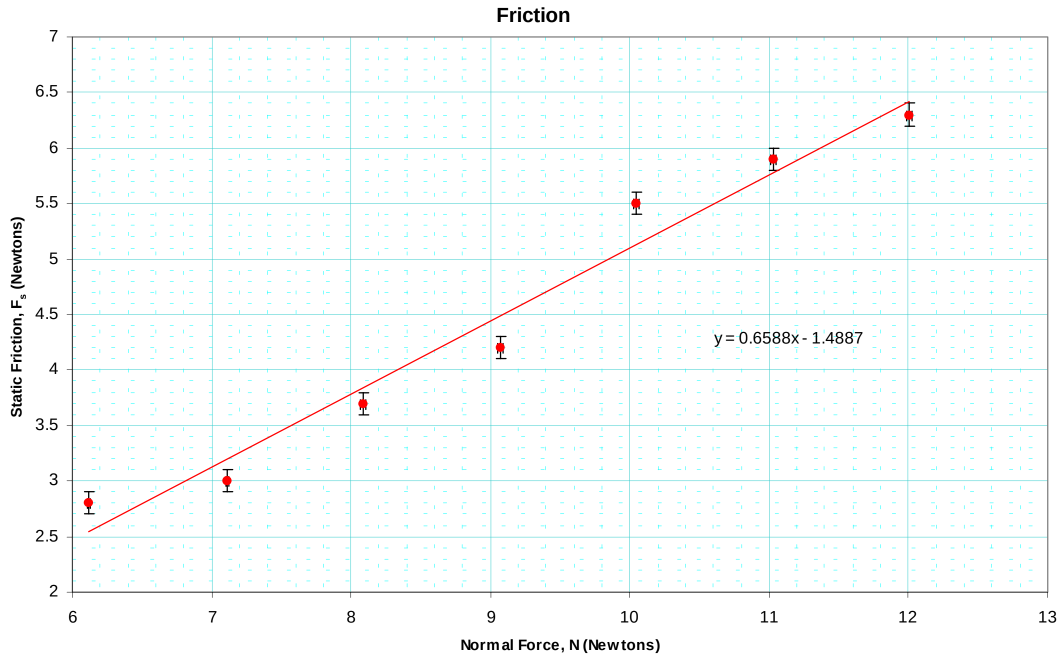
In the "Pressure of a Gas" graph the maximum deviation of a data point from the best-fit line is approximately the same size of the error bar. The assumption of uncertainty seems reasonable. We expect good results.

(b)



In the “Uniform Straight Line Motion” graph, the errorbars, and hence the uncertainties, have been overestimated by about a factor of two.

(c)

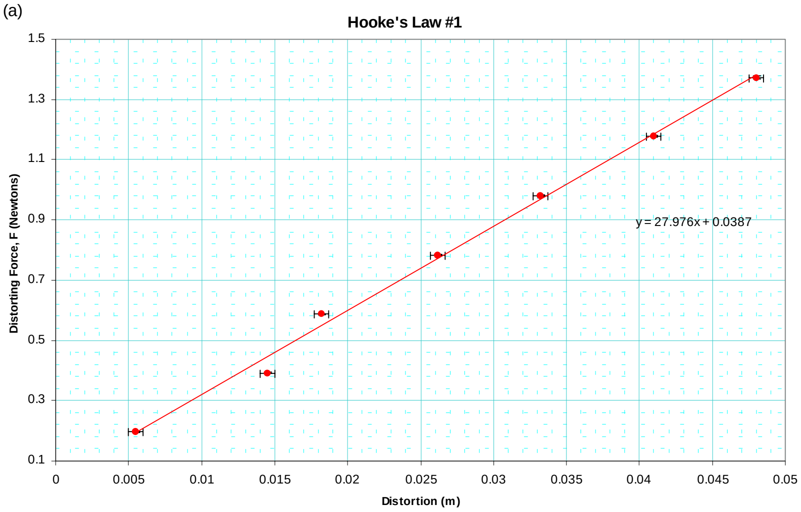


In the “Friction” graph, the errorbars, and hence the uncertainties, have been underrestimated by about a factor of about three.

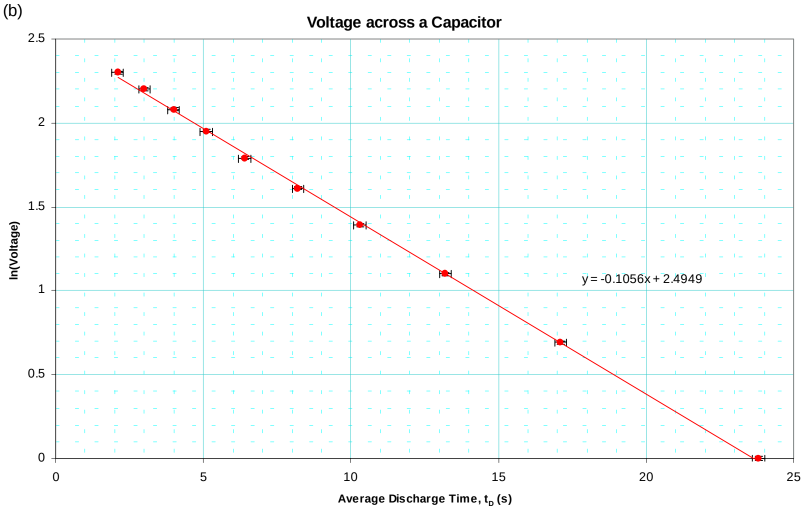
Exercises

For the following graphs, indicate if the uncertainties are correct, underestimated or overestimated. If underestimated or overestimated, say by how much.

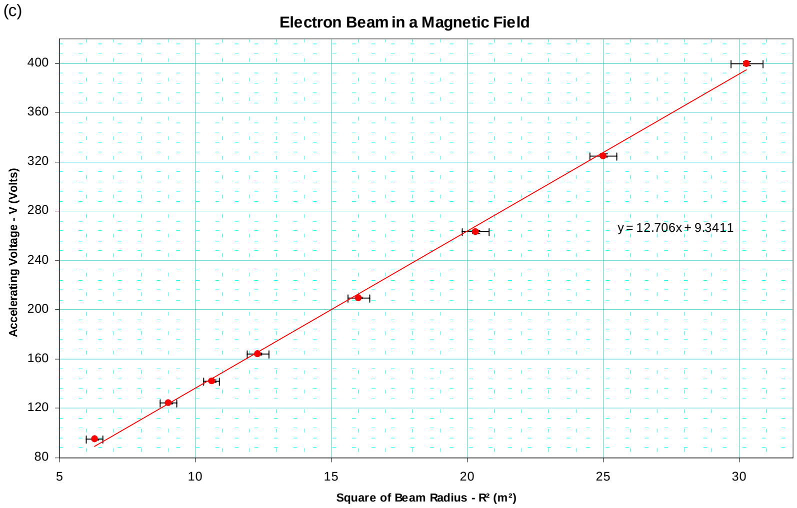
(a)



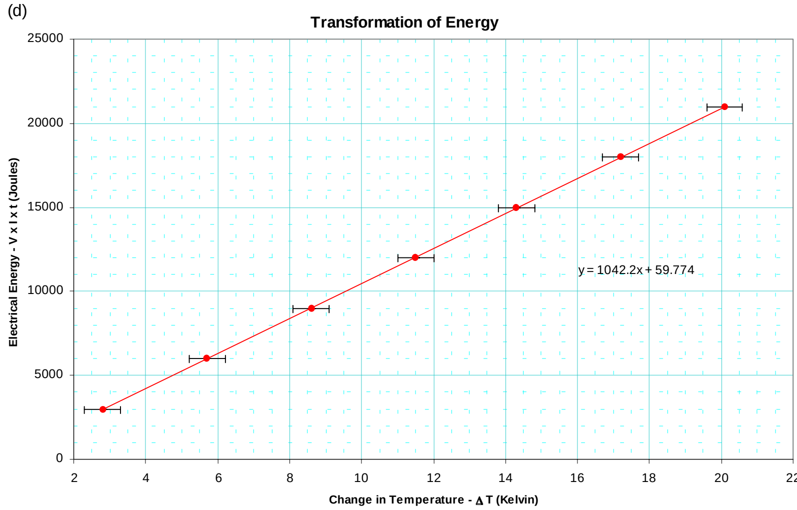
(b)


Voltage across a Capacitor
ln(Voltage)
2.5
2
1.5
1
0.5
0
0
5
10
15
20
25
Average Discharge Time, t_D (s)
y = -0.1056x + 2.4949

(c)


Electron Beam in a Magnetic Field
Accelerating Voltage - V (Volts)
400
360
320
280
240
200
160
120
80
5
10
15
20
25
30
Square of Beam Radius - R² (m²)
y = 12.706x + 9.3411

(d)


Transformation of Energy
Electrical Energy - V x I x t (Joules)
25000
20000
15000
10000
5000
0
2
4
6
8
10
12
14
16
18
20
22
Change in Temperature - Δ T (Kelvin)
y = 1042.2x + 59.774